

## Kutzbach Equation

### Planar Mechanism

Let

$n = \text{No. of links}$

$j = \text{No. of simple R pairs}$

No. of D.O.F of an unconnected rigid body in plane motion = 3

i.e. 2 translational & one rotational D.O.F.

No. of D.O.F of  $(n - 1)$  unconnected rigid body in plane motion =  $3(n - 1)$

Once two links are connected by an R pair,

No of D.O.F. lost = 2

No of D.O.F. left = 1

∴ No. of D.O.F of the mechanism,

$$F = 3(n - 1) - 2j$$

This equation is known as **Kutzbach Equation**.

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## Grubler criterion

### Planar Mechanism

If

$F = 1$ , the mechanism is a single D.O.F mechanism

$F = 2$ , the mechanism is a two D.O.F mechanism

$F = 0$ , the assembly is a structure

$F = -1$  or less, the assembly is a statically indeterminate structure

For a single D.O.F mechanism, putting  $F=1$  in the Kutzbach Eq., we get

$$2j - 3n + 4 + 0$$

This simple estimate of constrained movement is known as **Grubler criterion**.

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## Kutzbach Equation

### Planar Mechanism

#### Kutzbach Equation

$$F = 3(n - 1) - 2j$$

Since a Prismatic pair is a special case of a Revolute pair

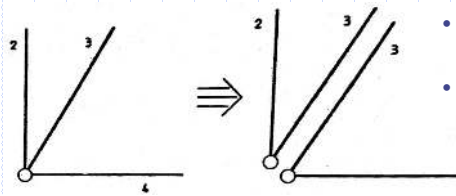
$\therefore j$  include prismatic pairs.

For higher order Pairs:

$$j = j_1 + 2j_2 + 3j_3 + \dots + ij_i$$

Where

$j_i$ : no of R pairs, each of which connects (i+1) links



- $j_2$  is equivalent to 2 simple R pairs
- Similarly,
- $j_i$  is equivalent to  $i$  nos. of simple R pairs

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## Kutzbach Equation

### Planar Mechanism

#### Kutzbach Equation for mechanism with higher pair

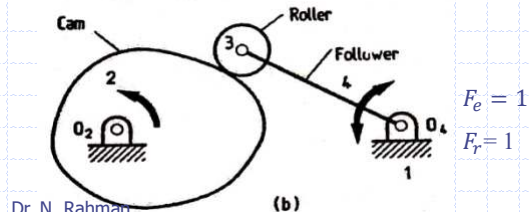
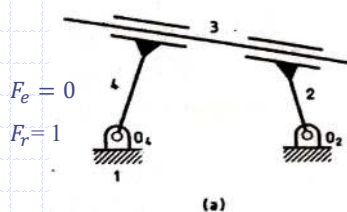
- A higher pair curtails 1 D.O.F

$\therefore$  D.O.F of mechanism having  $h$  nos. of higher pairs can be written as

$$F = 3(n - 1) - 2j - h$$

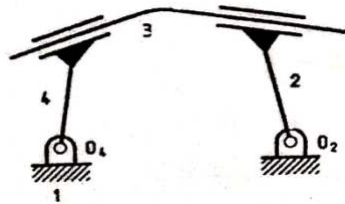
#### Effective D.O.F of mechanism with redundant degrees of freedom

$$F_e = 3(n - 1) - 2j - h - F_r$$



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## Kutzbach Equation



$$F_e = 0$$

$$F_r = 1$$

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## Kinematic Inversions

- Mechanisms that are derived from the same kinematic chain but have a different link fixed to ground.
- The relative motions of the links are the same in kinematic inversions (i.e., the motions at the joints are the same), but the absolute motions of the links are different, since they are being referenced to different links.

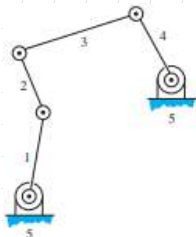


Figure (a)

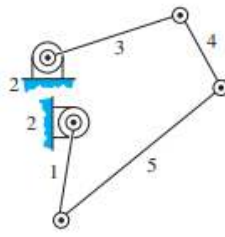


Figure (b): Two of the four possible kinematic inversions of the mechanism in Figure (a)

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## Kinematic Inversions

### Inversions Of 4 Bar Kinematic Chain

- If in a four bar kinematic chain all links are free, motion will be unconstrained.
- From a four link kinematic chain, four different mechanisms can be obtained by fixing each of the four links turn by turn.
- All these mechanisms are called inversions of the parent kinematic chain.
- By this principle of inversions of a four link chain, several useful mechanisms can be obtained.

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## Kinematic Inversions

### Inversions of Kinematic Chain with all the four kinematic pairs as revolute pairs

- All the four inversions of such a chain are identical.
- However, by suitably altering the proportions of lengths of links 1, 2, 3 and 4 respectively several mechanisms are obtained.

### Crank-rocker Mechanism

$$(l_1 + l_2) < (l_3 + l_4)$$

$$(l_2 + l_3) < (l_1 + l_4)$$

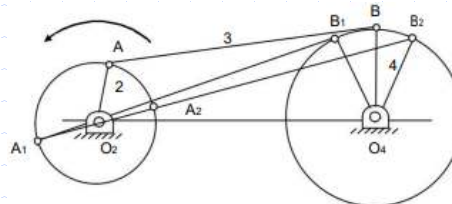


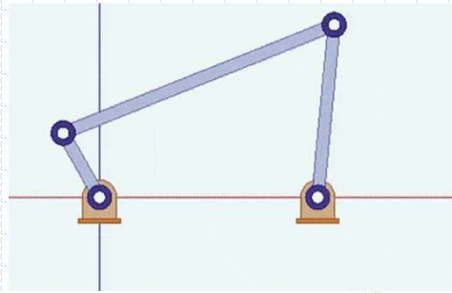
Figure: Crank-rocker Mechanism

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## Kinematic Inversions

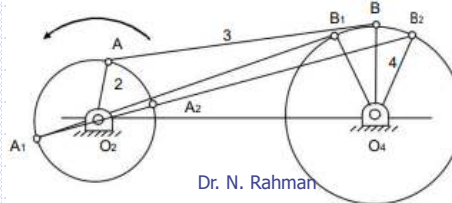
Inversions of Kinematic Chain with all the four kinematic pairs as revolute pairs

### Crank-rocker Mechanism



$$(l_1 + l_2) < (l_3 + l_4)$$

$$(l_2 + l_3) < (l_1 + l_4)$$

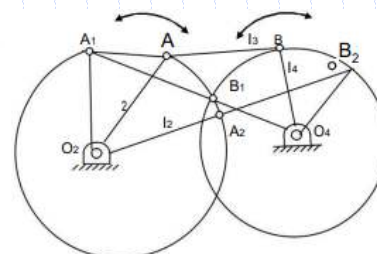
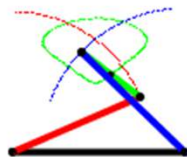


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## Kinematic Inversions

Inversions of Kinematic Chain with all the four kinematic pairs as revolute pairs

### Double Lever Mechanism or Rocker-Rocker Mechanism



$$(l_3 + l_4) < (l_1 + l_2)$$

$$(l_2 + l_3) < (l_1 + l_4)$$

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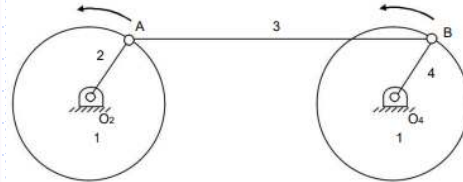
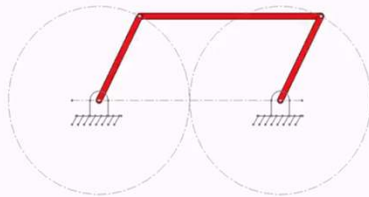
## Kinematic Inversions

### Inversions of Kinematic Chain with all the four kinematic pairs as revolute pairs

#### Double Crank Mechanism

- The links 2 and 4 of the double crank mechanism make complete revolutions. There are two forms of this mechanism.

#### ✓ Parallel Crank Mechanism



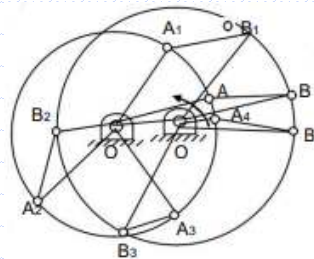
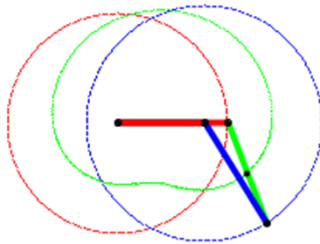
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## Kinematic Inversions

### Inversions of Kinematic Chain with all the four kinematic pairs as revolute pairs

#### Double Crank Mechanism

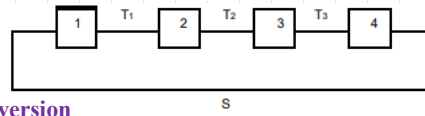
#### ✓ Drag Link Mechanism



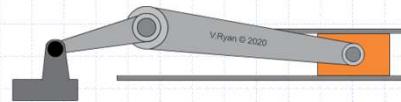
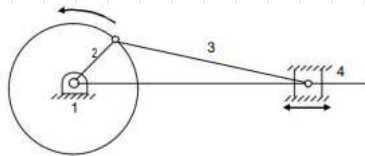
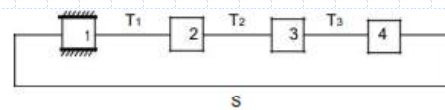
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## Kinematic Inversions

### Inversions of Kinematic Chain with three revolute pairs and one sliding pair



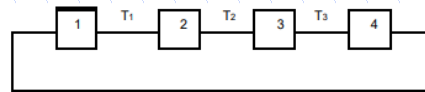
#### First Inversion



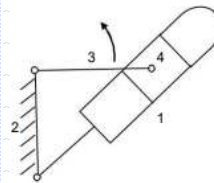
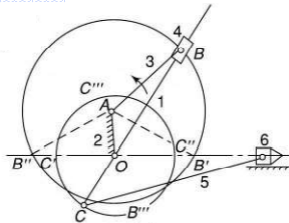
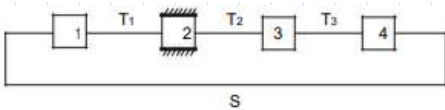
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## Kinematic Inversions

### Inversions of Kinematic Chain with three revolute pairs and one sliding pair



#### Second Inversion



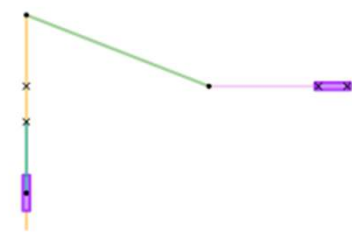
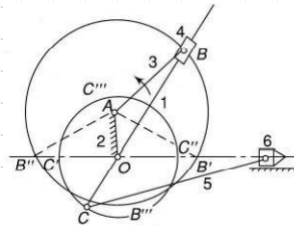
#### Whitworth Quick Return Motion Mechanism

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## Kinematic Inversions

### Inversions of Kinematic Chain with three revolute pairs and one sliding pair

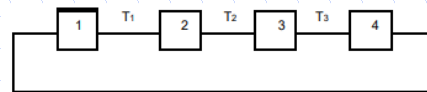
- The forward stroke starts when link 3 occupies position  $AB'$ . At that time, point C is at  $C'$ .
- The forward stroke ends when link 3 occupies position  $AB''$  and point C occupies position  $C''$ .
- The return stroke takes place when link 3 moves from position  $AB''$  to  $AB'$ .



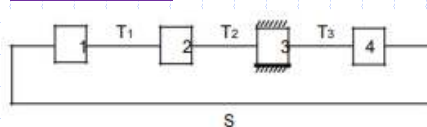
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## Kinematic Inversions

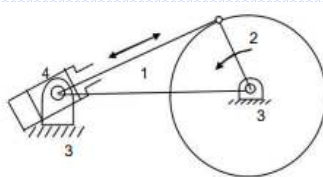
### Inversions of Kinematic Chain with three revolute pairs and one sliding pair



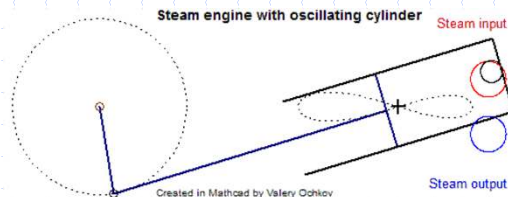
#### Third Inversion



- This inversion is obtained by fixing link 3.
- Link 1 works as a slider which slides in slotted or cylindrical link 4.
- Link 2 works as a crank.



(a) Oscillating Cylinder Engine



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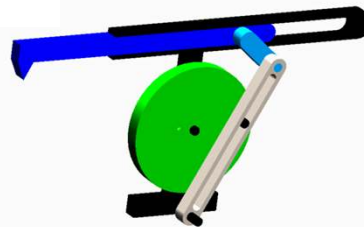
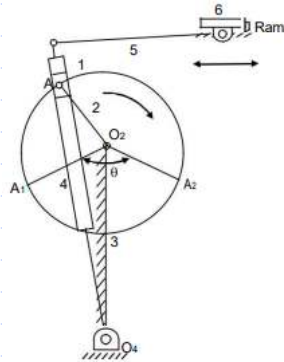
## Kinematic Inversions

### Third Inversion

- O<sub>2</sub>A<sub>1</sub> and O<sub>2</sub>A<sub>2</sub> are two positions of crank when link 4 will be tangential to the crank circle and corresponding to which ram will have extreme positions.
- When crank travels from position O<sub>2</sub>A<sub>1</sub> and O<sub>2</sub>A<sub>2</sub> forward stroke takes place.
- When crank moves further from position O<sub>2</sub>A<sub>2</sub> to O<sub>2</sub>A<sub>1</sub> return stroke takes place.

$$\text{Quick Return Ratio} = \frac{\text{Time for forward stroke}}{\text{Time for return stroke}}$$

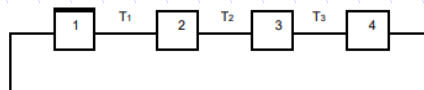
$$= \frac{(2\pi - \theta)}{\frac{\omega}{\theta}} = \frac{2\pi - \theta}{\theta}$$



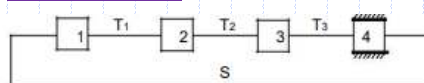
(b) Crank and Slotted Lever Mechanism

## Kinematic Inversions

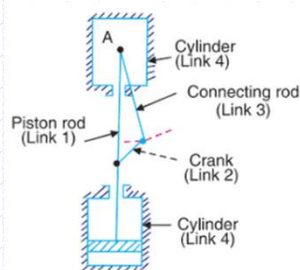
### Inversions of Kinematic Chain with three revolute pairs and one sliding pair



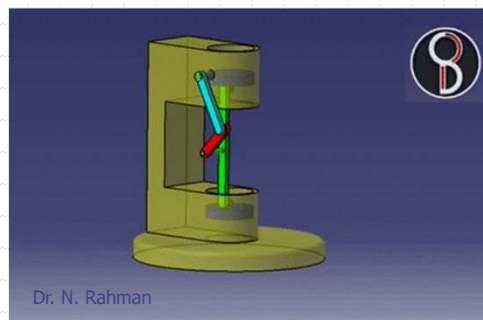
### Fourth Inversion



- This inversion is obtained by fixing link 4.
- The pendulum pump and hand pump are examples of this inversion.



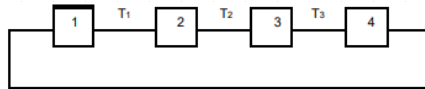
Pendulum Pump



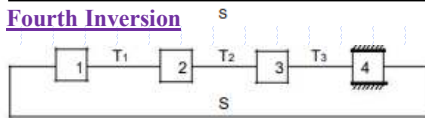
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## Kinematic Inversions

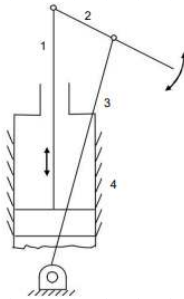
### Inversions of Kinematic Chain with three revolute pairs and one sliding pair



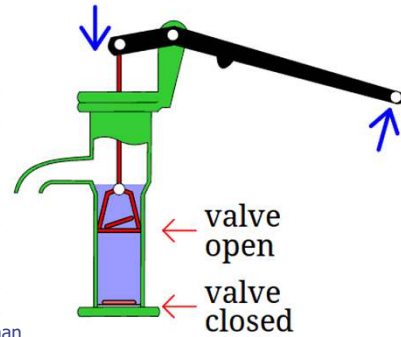
#### Fourth Inversion



Hand Pump



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## Kinematic Inversions

**P1: Discuss in detail the kinematic inversions of Double Slider Crank Chain.**

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