MECHANISMS WITH LOWER PAIR

STRAIGHT-LINE MECHANISMS

Paucellier Mechanism

A Paucellier mechanism consists of eight links such that

$OA = OQ;$ $AB = AC$

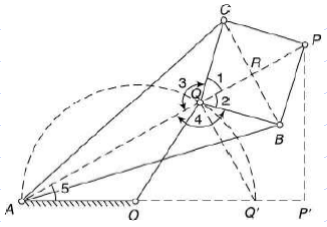
$BP = PC = CQ = QB$

OA: Fixed link

OQ: Rotating Link

As the link OQ moves around O, P moves in a straight line perpendicular to OA.

All the joints are pin-jointed.



Since $BPCQ$ is a rhombus,

QP always bisects the angle BQC,

i.e.,

$$\angle 1 = \angle 2 \qquad \text{(i)}$$

in all the positions

Also, in Δs AQC and AQB,

AQ is common,

$AC = AB$

$QC = QB$

$\therefore$ Δs are congruent in all positions.

$\Rightarrow$ $\angle 3 = \angle 4$ (ii)

Adding (i) and (ii),

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

But $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360^o$

$\therefore$ $\angle 1 + \angle 3 = \angle 2 + \angle 4 = 180^o$

Or A,Q and P lie on a straight line.

Let $PP' \perp AO$

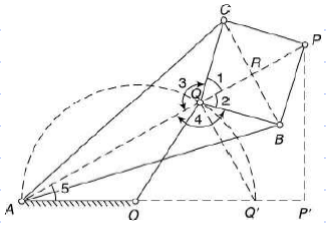
$$\Delta AQQ' \sim \Delta AP'P$$

$$\therefore \frac{AQ}{AP'} = \frac{AQ'}{AP}$$

or $AQ'.AP' = AQ.AP$

STRAIGHT-LINE MECHANISMS

Paucellier Mechanism



or $AQ'.\ AP' = AQ.AP$

$= (AR - RQ)\ (AR + RP)$
$= (AR - RQ)\ (AR + RQ)$
$= (AR)^2 - (RQ)^2$
$= [(AC)^2 - (CR)^2) - [(CQ)^2 - (CR)^2]$

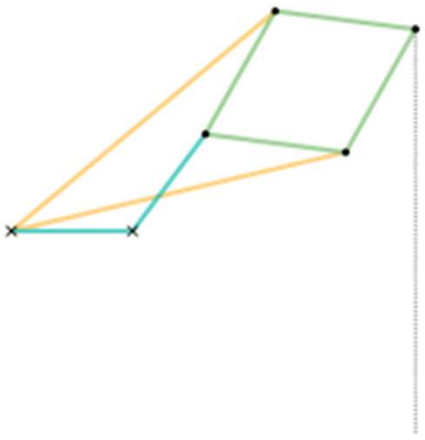
$$AP' = \frac{(AC)^2 - (CQ)^2}{AQ'}$$

= constant, as AC, CQ and AQ' are always fixed

This means that the projection of P on AO produced is constant for all the configurations. or P moves in a straight line perpendicular to AO.

STRAIGHT-LINE MECHANISMS

Paucellier Mechanism



STRAIGHT-LINE MECHANISMS

Hart Mechanism

- This mechanism requires only six links as compared with the eight links required by the Peaucellier mechanism.

$AB = CD;\quad AD = BC$ and $OE = OQ$

OE is the fixed link and OQ, the rotating link.

- ABCD is a trapezium

$$\frac{AE}{AB} = \frac{AQ}{AD} = \frac{CP}{CB} \qquad \text{(i)}$$

- OQ rotates about O, P moves in a line perpendicular to EO produced.

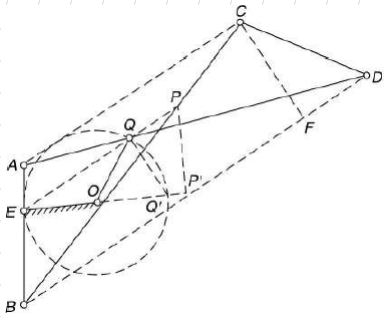
In ΔABD $\dfrac{AE}{AB} = \dfrac{AQ}{AD}$ (Given)

$\therefore$ EQ is parallel to BD and thus parallel to AC.

In ΔABC $\dfrac{AE}{AB} = \dfrac{CP}{CB}$ (Given)

$\therefore$ EP is parallel to AC and thus parallel to BD.

$\Rightarrow$ EQP is a straight line



STRAIGHT-LINE MECHANISMS

Hart Mechanism

Δs AEQ and ABD are similar ($\because$ $EQ \parallel BD$).

$$\therefore \frac{EQ}{BD} = \frac{AE}{AB} \text{ or } EQ = BD \times \frac{AE}{AB} \qquad \text{(ii)}$$

Δs BEP and BAC are similar ($\because$ $EP \parallel AC$).

$$\therefore \frac{EP}{AC} = \frac{BE}{BA} \text{ or } EP = AC \times \frac{BE}{AB} \qquad \text{(iii)}$$

Δs EQQ' and $EP'P$ are similar

$\because$ $\angle QEQ'$ or $\angle PEP'$ is common

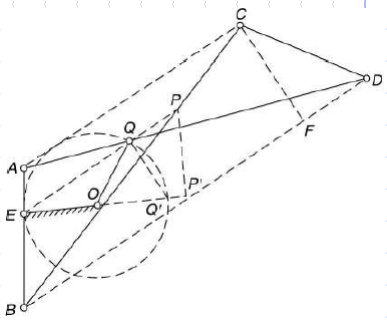
and $\angle EQQ' = \angle QP'P = 90°$.

$$\therefore \frac{EQ}{EP'} = \frac{EQ'}{EP}$$

$$EP' = \frac{AE \times BE}{(EQ')(AB)^2}[(BD)(AC)] \qquad \text{[from (ii) and (iii)]}$$

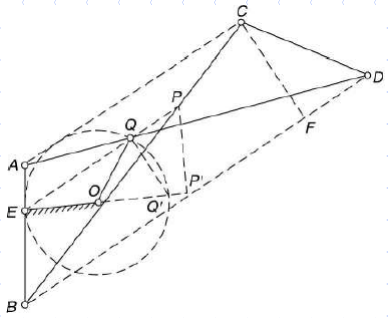
$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BF + FD)(BF - FD)\right]$$

$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BF)^2 - (FD)^2\right]$$



STRAIGHT-LINE MECHANISMS

Hart Mechanism



$$EP' = \frac{AE \times BE}{(EQ')(AB)^2}\left[\{(BC)^2 - (CF)^2\} - \{(CD)^2 - (CF)^2\}\right]$$

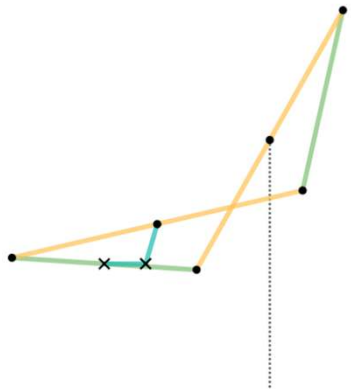
$$= \frac{AE \times BE}{(EQ')(AB)^2}\left[(BC)^2 - (CD)^2\right]$$

= constant, as all the parameters are fixed.

∴ The projection of P on EO produced is always the same point or P moves in a straight line perpendicular to EO.

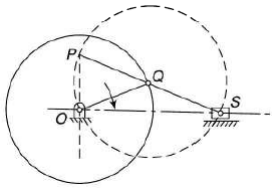
STRAIGHT-LINE MECHANISMS

Hart Mechanism



STRAIGHT-LINE MECHANISMS

Scott-Russel Mechanism



A Scott–Russel mechanism consists of three movable links; *OQ*, *PS* and slider *S* which moves along *OS*. *OQ* is the crank.

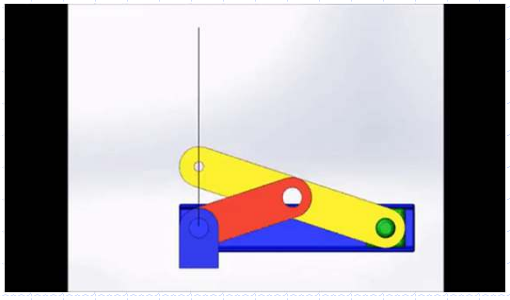
The links are connected in such a way that

$$QO = QP = QS$$

P moves in a straight line perpendicular to *OS* at *O* as the slider *S* moves along *OS*.

STRAIGHT-LINE MECHANISMS

Scott-Russel Mechanism



STRAIGHT-LINE MECHANISMS

P2: Discuss in detail three Approximate Straight Line Motion Mechanisms.