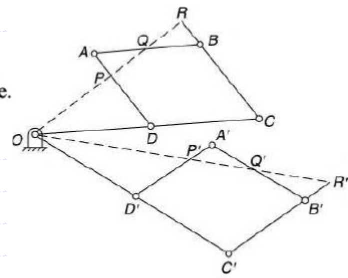
Pantograph

- A pantograph is an instrument used to reproduce to an enlarged or a reduced scale and as exactly as possible the path described by a given point.
  - The four links of a pantograph are arranged in such a way that a parallelogram ADCD is formed.
  - AB = DC and BC = AD
  - O,P,Q and R lie on links CD, DA, AB and BC respectively such that OPQR is a straight line.
  - ABCD is the initial assumed position.
  - $A'B'C'D'$ is the new position.



In ΔODP and OCR,
O, P and R lie on a straight line and thus OP and OR coincide.
$\angle DOP = \angle COR$ (common angle)
$\angle ODP = \angle OCR$ ($\because DP \,||\, CR$)

Therefore, the Δs are similar and

$$\frac{OD}{OC} = \frac{OP}{OR} = \frac{DP}{CR}$$

Now, $A'B' = AB = DC = D'C'$
And $B'C' = BC = AD = A'D'$
Therefore, $A'B'C'D'$ is again a parallelogram.

In Δs $OD'P'$ and $OC'R'$,

$$\frac{OD'}{OC'} = \frac{OD}{OC} = \frac{DP}{CR}$$
$$= \frac{D'P'}{C'R'}$$

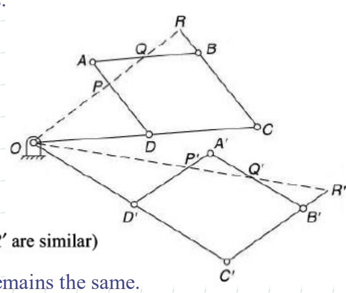
and,
$\angle OD'P' = \angle OC'R'$ ($D'P' \,||\, C'B'$ as $A'B'C'D'$ is a || gm)

Thus the triangles $OD'P'$ and $OC'R'$ are similar triangles.

$\therefore$ $\angle D'OP' = \angle C'OR'$

or O, P' and R' lie on a straight line.
Now

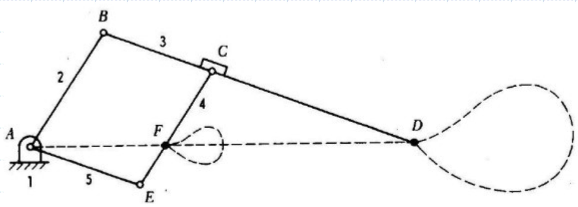
$$\frac{OP}{OR} = \frac{OD}{OC}$$
$$= \frac{OD'}{OC'}$$
$$= \frac{OP'}{OR'} \quad (\because \Delta s\ OD'P' \text{ and } OC'R' \text{ are similar})$$



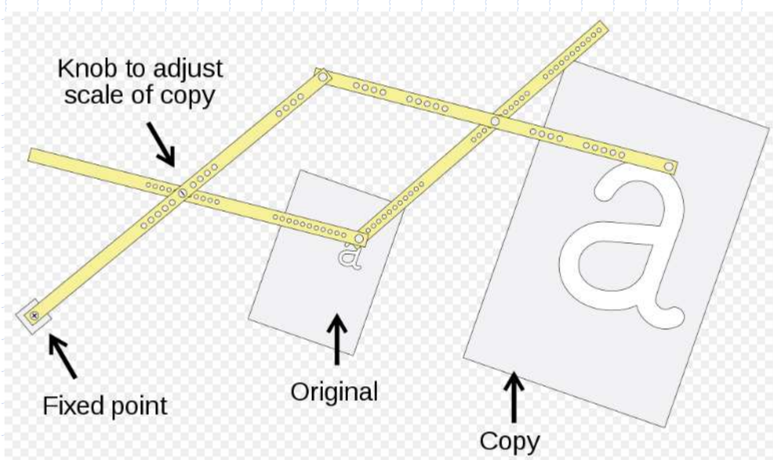
$\therefore$The ratio of distances of P and R from the fixed point remains the same.
This will be true for all the positions of the links.
Thus, P and R will trace exactly similar paths.

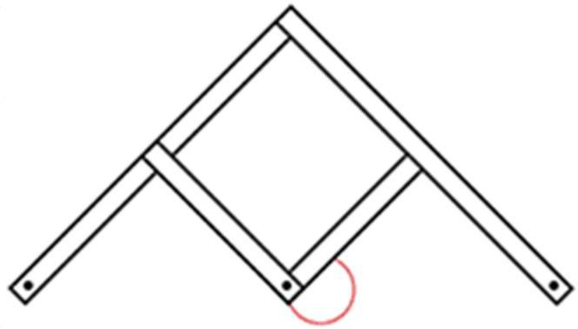
Uses of Pantograph

- A pantograph is an instrument used to reproduce to an enlarged or a reduced scale and as exactly as possible the path described by a given point.
- A pantograph is mostly used for the reproduction of plane areas and figures such as maps, plans etc., on enlarged or reduced scales.
- It is, sometimes, used as an indicator rig in order to reproduce to a small scale the displacement of the crosshead and therefore of the piston of a reciprocating steam engine. It is also used to guide cutting tools.
- A modified form of pantograph is used to collect power at the top of an electric locomotive.



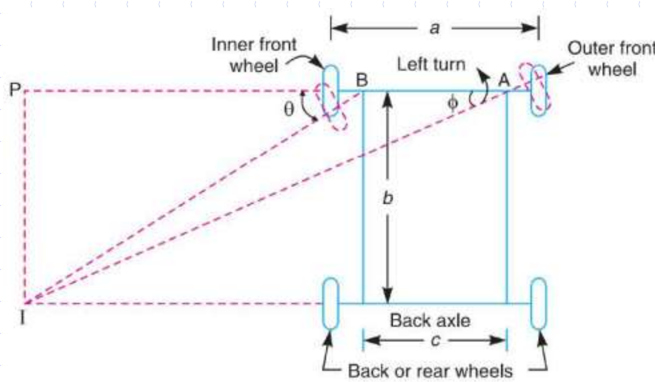
Uses of Pantograph





STEERING GEAR MECHANISM

- The steering gear mechanism is used for changing the direction of two or more of the wheel axles with reference to the chassis, so as to move the automobile in any desired path.
- Usually the two back wheels have a common axis, which is fixed in direction with reference to the chassis and the steering is done by means of the front wheels.



STEERING GEAR MECHANISM

- Thus, the condition for correct steering is that all the four wheels must turn about the same instantaneous centre.
- The axis of the inner wheel makes a larger turning angle θ than the angle ϕ subtended by the axis of outer wheel.

Let a = Wheel track,

b = Wheel base, and

c = Distance between the pivots A and B of the front axle.

Now from triangle IBP,

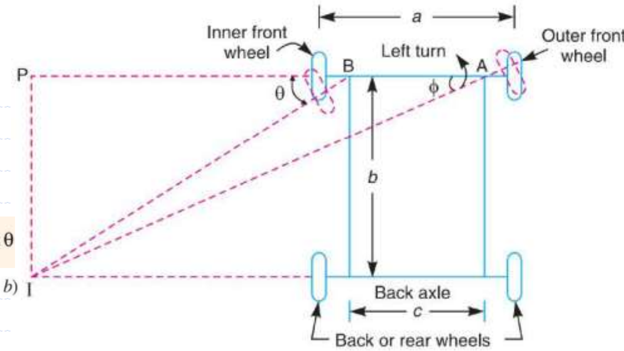
$$\cot\theta = \frac{BP}{IP}$$

and from triangle IAP,

$$\cot\phi = \frac{AP}{IP} = \frac{AB + BP}{IP}$$

$$= \frac{AB}{IP} + \frac{BP}{IP} = \frac{c}{b} + \cot\theta \quad ...(\because IP = b)$$

$$\therefore \cot\phi - \cot\theta = c / b$$

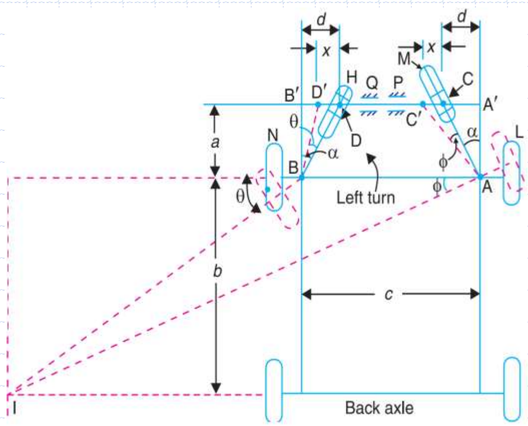


- This is the fundamental equation for correct steering. If this condition is satisfied, there will be no skidding of the wheels, when the vehicle takes a turn.

DAVIS STEERING GEAR MECHANISM

- It is an exact steering gear mechanism. The slotted links AM and BH are attached to the front wheel axle, which turn on pivots A and B respectively.
- The rod CD is constrained to move in the direction of its length, by the sliding members at P and Q. These constraints are connected to the slotted link AM and BH by a sliding and a turning pair at each end.

a = Vertical distance between AB and CD,
b = Wheel base,
$2d$ = Difference between AB and CD ,
c = Distance between the pivots A and B of the front axle.
x = Distance moved by C to C' = $CC' = DD'$, and
α = Angle of inclination of the links AC and BD, to the vertical.



DAVIS STEERING GEAR MECHANISM

From triangle $AA'C'$,

$$\tan(\alpha + \phi) = \frac{A'C'}{AA'} = \frac{d + x}{a}$$

From triangle $AA'C$,

$$\tan \alpha = \frac{A'C}{AA'} = \frac{d}{a}$$

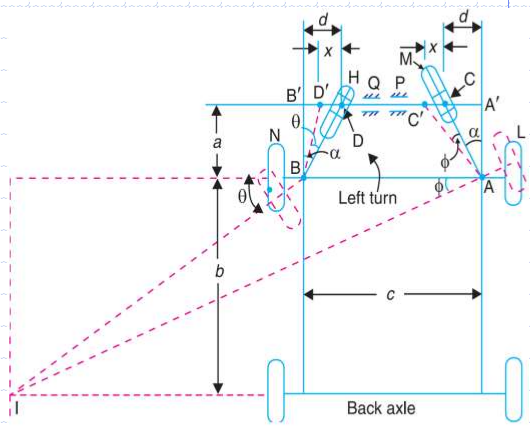
From triangle $BB'D'$,

$$\tan(\alpha - \theta) = \frac{B'D'}{BB'} = \frac{d - x}{a}$$

We know that

$$\tan(\alpha + \phi) = \frac{\tan\alpha + \tan\phi}{1 - \tan\alpha.\tan\phi}$$

$$\frac{d + x}{a} = \frac{d/a + \tan\phi}{1 - d/a \times \tan\phi} = \frac{d + a\tan\phi}{a - d\tan\phi}$$



DAVIS STEERING GEAR MECHANISM

$$\frac{d + x}{a} = \frac{d/a + \tan\phi}{1 - d/a \times \tan\phi} = \frac{d + a\tan\phi}{a - d\tan\phi}$$

$$\Rightarrow \quad \tan\phi = \frac{a.x}{a^2 + d^2 + d.x}$$

Similarly, from

$$\tan(\alpha - \theta) = \frac{d - x}{a}, \quad \text{we get}$$

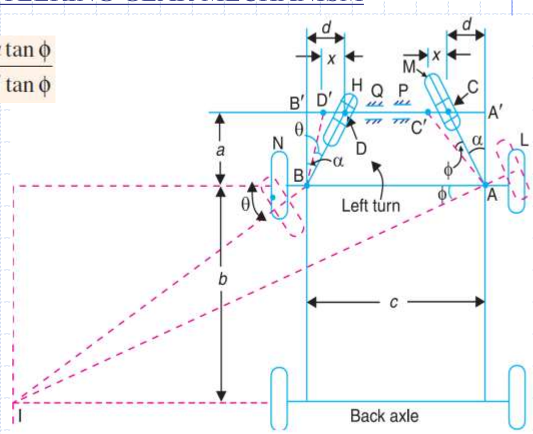
$$\tan\theta = \frac{ax}{a^2 + d^2 - d.x}$$

We know that for correct steering,

$$\cot\phi - \cot\theta = \frac{c}{b} \quad \text{or}$$

$$\frac{1}{\tan\phi} - \frac{1}{\tan\theta} = \frac{c}{b} \quad \text{or} \quad \frac{a^2 + d^2 + d.x}{a.x} - \frac{a^2 + d^2 - d.x}{a.x} = \frac{c}{b}$$

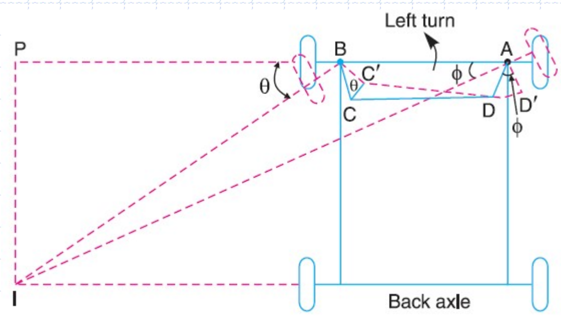
$$\Rightarrow \quad \tan\alpha = \frac{c}{2b}$$



- The usual value of $\frac{c}{2b}$ is between 0.4 to 0.5 and that of α from 11 to 14 degrees.

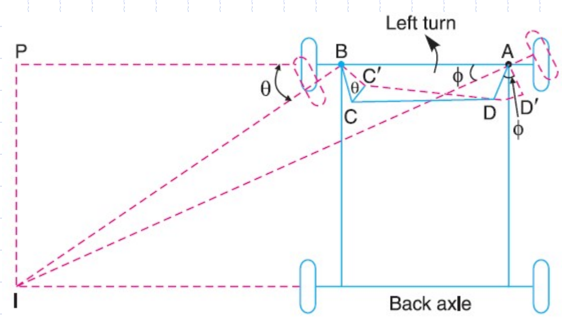
ACKERMANN STEERING GEAR MECHANISM

- The Ackerman steering gear **mechanism is much simpler than Davis gear.** The difference between the Ackerman and Davis steering gears are:
  - ✓ The whole mechanism of the **Ackerman steering gear is on back of the front wheels; whereas in Davis steering gear, it is in front of the wheels.**
  - ✓ The Ackerman **steering gear consists of turning pairs,** whereas Davis steering gear consists of sliding members.
- The **shorter links BC and AD are of equal length** and are connected by hinge joints with front wheel axles.
- The **longer links AB and CD are of unequal length.**

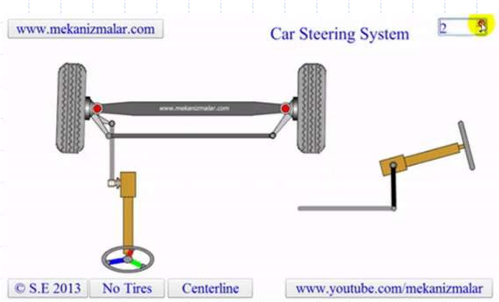


ACKERMANN STEERING GEAR MECHANISM

- The following are the only three positions for correct steering.
  - ✓ When the vehicle moves along a straight path, the longer links A B and CD are parallel and the shorter links BC and AD are equally inclined to the longitudinal axis of the vehicle, as shown by firm lines in Fig. 2.12.
  - ✓ When the vehicle is steering to the left, the position of the gear is shown by dotted lines in Fig. In this position, the lines of the front wheel axle intersect on the back wheel axle at I, for correct steering.
  - ✓ When the vehicle is steering to the right, the similar position may be obtained.




ACKERMANN STEERING
4-BAR STEERING
ACKERMANN STEERING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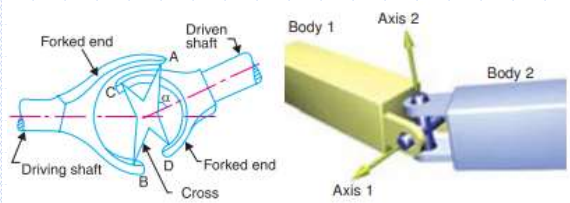
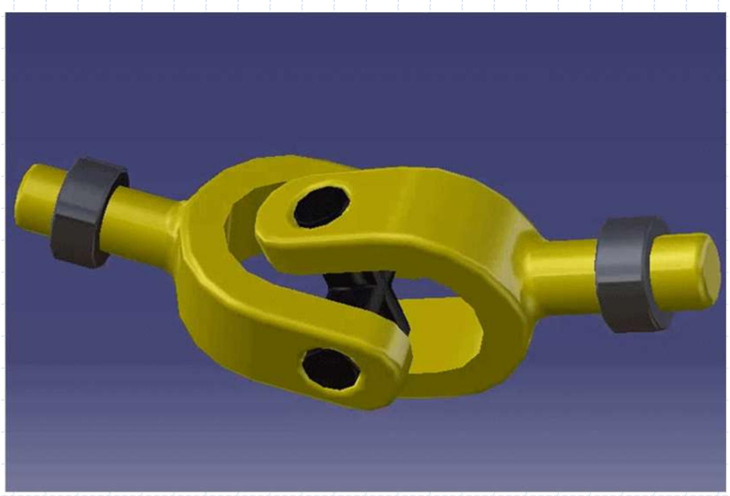
Hook's joint

- ➢ A Hooke's joint is used to connect two non parallel and intersecting shafts.
- ➢ It is also used for shafts with angular misalignment.
- ➢ The driving shaft rotates at a uniform angular speed whereas the driven shaft rotates at a continuously varying angular speed.

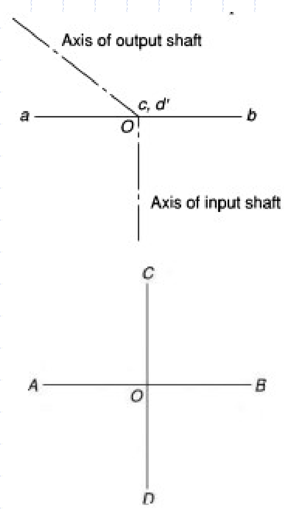


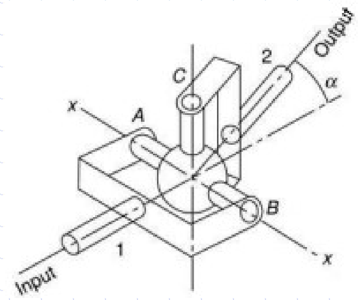
Applications:

- ✓ Used in automobiles where it is used to transmit power from the gear box of the engine to the rear axle.
- ✓ Used for transmission of power to different spindles of multiple drilling machine.
- ✓ Used as a knee joint in milling machines.



Hook's joint





Hook's joint

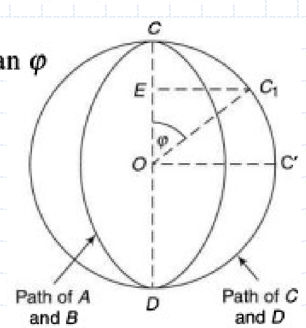
$$\frac{\tan \varphi}{\tan \theta} = \frac{EC_1' \,/\, EO}{EC_1 \,/\, EO} = \frac{EC_1'}{EC_1}$$

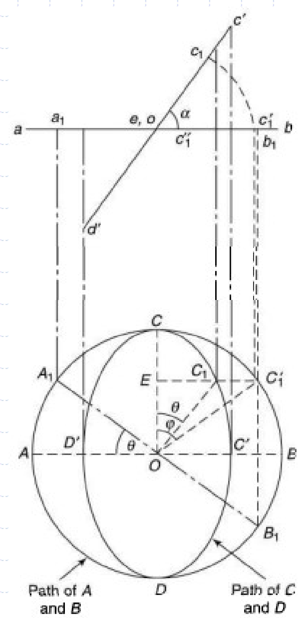
$$= \frac{ec_1'}{ec_1''} \quad \text{(from top view)}$$

$$= \frac{ec_1}{ec_1''} = \frac{1}{ec_1''/\,ec_1} = \frac{1}{\cos a}$$

$\Longrightarrow$

$$\tan \theta = \cos \alpha \tan \varphi$$





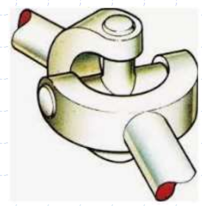
Hook’s joint

Angular Velocity Ratio

ω_1 : Angular velocity of driving shaft

ω_2 : Angular velocity of driven shaft

$\omega_1 = \frac{d\theta}{dt}$ $\quad \omega_2 = \frac{d\varphi}{dt}$



We have

$$\omega_2 = \frac{d\varphi}{dt}$$

Differentiating the above equation w.r.t. time *t*

$$\sec^2\theta \frac{d\theta}{dt} = \cos\alpha \sec^2\varphi \frac{d\varphi}{dt} \quad \text{or}$$

$$\frac{\omega_2}{\omega_1} = \frac{1}{\cos^2\theta\cos\alpha(1+\tan^2\varphi)} = \frac{1}{\cos^2\theta\cos\alpha\left(1+\frac{\tan^2\theta}{\cos^2\alpha}\right)} = \frac{\cos\alpha}{\cos^2\theta(1-\sin^2\alpha)+\sin^2\theta}$$

$$= \frac{\cos\alpha}{\cos^2\theta - \cos^2\theta\sin^2\alpha + \sin^2\theta} = \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}$$

Hook’s joint

Angular Velocity Ratio

1) For equal velocities of the driving and driven shafts, $\frac{\omega_2}{\omega_1} = 1$

But $\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta}$

$$\therefore \quad \frac{\cos\alpha}{1-\sin^2\alpha\cos^2\theta} = 1$$

or $\quad \cos^2\theta = \frac{1-\cos\alpha}{\sin^2\alpha} = \frac{1}{1+\cos\alpha} = \frac{\cos^2\theta\left(\frac{\sin^2\theta}{\cos^2\theta}+1\right)}{1+\cos\alpha}$

or $\quad \frac{\sin^2\theta}{\cos^2\theta} + 1 = 1 + \cos\alpha$

or $\quad \tan\theta = \pm\sqrt{\cos\alpha}$

- Therefore velocities of the driven and driving shafts are equal once in all the four quadrants for particular values of θ if α is constant

Hook's joint

Angular Velocity Ratio

2) For minimum velocity ratio, $\frac{\omega_2}{\omega_1}$

$(1 - \sin^2\alpha\cos^2\theta)$ needs to be maximum $\quad \because \frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1 - \sin^2\alpha\cos^2\theta}$

This happens when $\cos^2\theta$ is minimum

or $\quad \theta = 90° \text{ or } 270°$

Then $\quad \frac{\omega_2}{\omega_1} = \cos\alpha$

3) For maximum velocity ratio, $\frac{\omega_2}{\omega_1}$

$(1 - \sin^2\alpha\cos^2\theta)$ needs to be minimum $\quad \because \frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1 - \sin^2\alpha\cos^2\theta}$

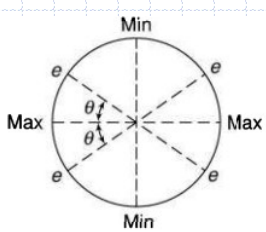
This happens when $\cos^2\theta$ is maximum

or $\quad \theta = 0° \text{ or } 180°$

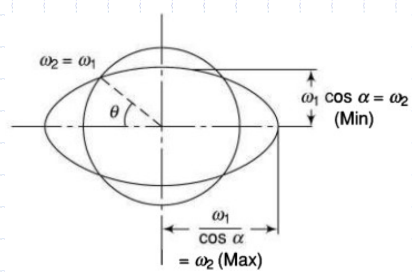
Then $\quad \frac{\omega_2}{\omega_1} = \frac{1}{\cos\alpha}$

Hook's joint

Angular Velocity Ratio



Variation in the speed of driven shaft



Polar velocity Diagram

$$\text{Maximum variation} = \frac{\omega_{2\,max} - \omega_{2\,min}}{\omega_{mean}} = \frac{\omega_1/\cos\alpha - \omega_1\cos\alpha}{\omega_1}$$

$$= \tan\alpha\sin\alpha$$

$$\approx \alpha^2 \quad \text{(for small } \alpha\text{)}$$

Hook's joint

Angular Acceleration

ω_1: constant

$$\frac{\omega_2}{\omega_1} = \frac{\cos\alpha}{1 - \sin^2\alpha\cos^2\theta}$$

For finding the acceleration, differentiating the above equation w.r.t.

$$\frac{d\omega_2}{dt} = \omega_1 \frac{d}{dt}\left(\frac{\cos\alpha}{1 - \sin^2\alpha\cos^2\theta}\right)$$

$$\text{acceleration} = \omega_1 . \frac{d\theta}{dt}\frac{d}{d\theta}\left(\frac{\cos\alpha}{1 - \sin^2\alpha\cos^2\theta}\right)$$

$$= \omega_1^2 \cos\alpha \frac{d}{d\theta}(1 - \sin^2\alpha\cos^2\theta)^{-1}$$

$$= \frac{-\omega_1^2 \cos\alpha \sin^2\alpha \sin 2\theta}{(1 - \sin^2\alpha\cos^2\theta)^2}$$

Hook's joint

Angular Acceleration

Acceleration is minimum or maximum when $\frac{d\,(acc)}{d\theta} = 0$
Which gives

$$\cos 2\theta \approx \frac{2\sin^2\alpha}{2 - \sin^2\alpha}$$

Double Hook's joint

A universal joint, consisting of two Hooke's joints with an intermediate shaft, that eliminates variations in angular displacement and velocity between the driving and driven shafts

- This speed of the driving and driven shaft is constant
- This joint gives a velocity ratio equal to unity, if
  1. The axes of the driving and driven shafts are in the same plane, and
  2. The driving and driven shafts make equal angles with the intermediate shaft
  3. The two forks at the ends of intermediate shaft lie in the same plane.

$\tan\theta = \cos\alpha \tan\gamma$

$\tan\varphi = \cos\alpha \tan\gamma$

$\therefore\ \theta = \varphi$