# MEC2120
# Kinematics of Machines

## Unit 2

# Kinematic Analysis

- Kinematics is the study of displacement, rotation, speed, velocity and acceleration of each link at various positions during the operating cycle.
- Using these information, a designer can compute forces and thereby dimensions of all the links.
- In kinematic analysis, the motion of a point on the link relative to the fixed frame of reference is called absolute motion. However, the motion of a point relative to some other moving link or moving frame of reference is said to be relative motion.

# Velocity Analysis

- The change of position of a link with reference to some fixed frame of coordinates is called displacement.
- The rate of change of displacement of a link with reference to time, i.e. the time derivative of displacement, is commonly referred as velocity of link.
- Depending upon the type of motion, the velocity is classified into two types, namely linear velocity and angular velocity.

Linear velocity: $v = \frac{ds}{dt}$

Angular velocity: $\omega = \frac{d\theta}{dt}$

In kinematic analysis, the velocities of various links can be determined by the following methods:

(i) Relative velocity method or velocity polygon method
(ii) Instantaneous center method
(iii) Analytical method

## Relative velocity method

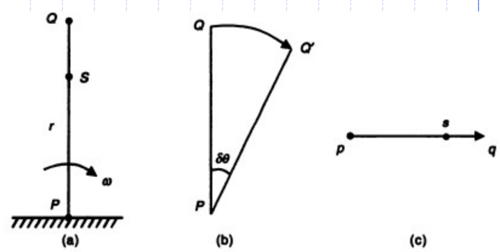
$$v_{qp} = \frac{\delta\theta}{\delta t} \times r$$

$$v_{qp} = \frac{d\theta}{dt} \times r \text{ when } \delta t \to 0$$

$$v_{qp} = \omega r$$

$$v_{sp} = \omega \cdot SP$$

$$\frac{v_{sp}}{v_{qp}} = \frac{\overrightarrow{ps}}{\overrightarrow{pq}} = \frac{\omega \times SP}{\omega \times QP} = \frac{SP}{QP}$$



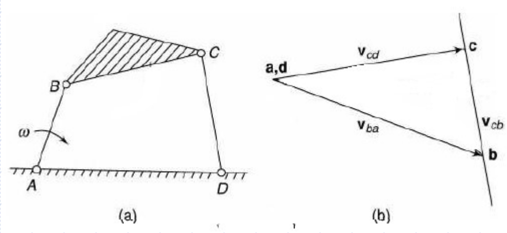

- Hence the point s divides the vector pq in the same ratio as the point S divides the link PQ.
- Thus, this law of proportionality is useful in drawing the velocity polygon and finding the relative velocities of points on the link.

# Four Link Mechanism

◆ **Four Bar Mechanism**

**AD: Fixed Link, AB: Crank, CD: Rocker**

$\omega$: ***Angular Speed of AB (Clockwise)***




$\vec{V}_{ca} = \vec{V}_{ba} + \vec{V}_{cb}$ or $\vec{V}_{cd} = \vec{V}_{ba} + \vec{V}_{cb}$ or $\overrightarrow{dc} = \overrightarrow{ab} + \overrightarrow{bc}$

**Where,** $\vec{V}_{ba}$ **or** $\overrightarrow{ab} = \omega AB$; $\perp$ **to** $AB$

$\vec{V}_{cb}$ **or** $\overrightarrow{bc}$ **is unknown in magnitude;** $\perp$ **to** $BC$

$\vec{V}_{cd}$ **or** $\overrightarrow{dc}$ **is unknown in magnitude;** $\perp$ **to** $DC$

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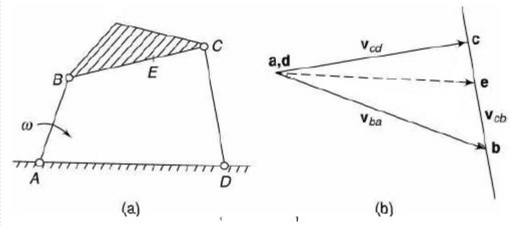
# Four Link Mechanism

◆ **Four Bar Mechanism**

**Velocity of Intermediate Point (Point E on Link $BC$)**




$$\frac{be}{bc} = \frac{BE}{BC}$$

**Here,** $\overrightarrow{ae}$: *Absolute Velocity of Point E on Link BC*

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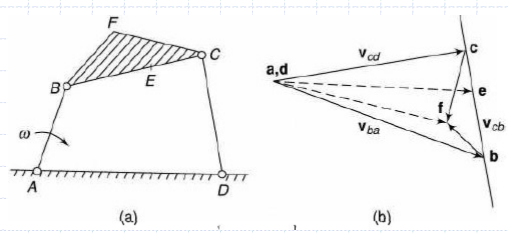
# Four Link Mechanism

◆ **Four Bar Mechanism**

**Velocity of Offset Point (Point F on Link $BC$)**




$$\vec{V}_{ba} + \vec{V}_{fb} = \vec{V}_{cd} + \vec{V}_{fc}$$

$$\overrightarrow{ab} + \overrightarrow{bf} = \overrightarrow{dc} + \overrightarrow{cf}$$

$\vec{V}_{fb} \perp$ **to** $BF$ $\quad\quad$ $\vec{V}_{fc} \perp$ **to** $CF$

**Here,** $\overrightarrow{af}$: *Absolute Velocity of Point F on Link BC*

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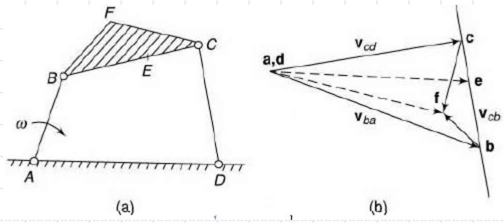
# Four Link Mechanism

- **Four Bar Mechanism**

**Velocity of Offset Point (Point F on Link $BC$)**




**Velocity Image:**

$\triangle bfc \sim \triangle BFC$

**Where,**

$bf \perp$ to $BF$ $\quad fc \perp$ to $FC$ **and** $cb \perp$ to $CB$

**Here,** $\triangle bfc$ **is called Velocity Image**

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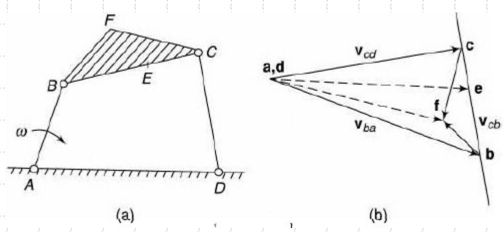
# Four Link Mechanism

- **Four Bar Mechanism**

**Angular Velocity of Links**




- **Link $BC$**

$\omega_{CB} = \frac{v_{cb}}{CB}$ **( C.C.W )**

- **Link $CD$**

$\omega_{CD} = \frac{v_{cd}}{CD}$ **( C.W )**

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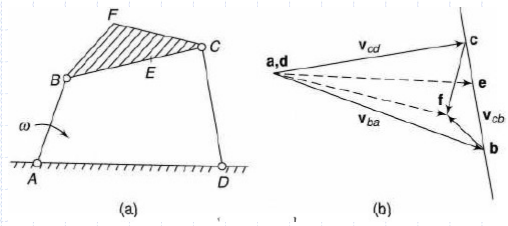
# Four Link Mechanism

- **Four Bar Mechanism**

**Velocity of Rubbing**



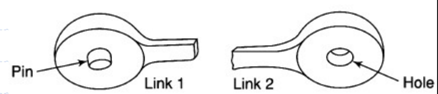

- **Pin at $A$**

  **Velocity of Rubbing** $= r_A \omega_{AB} = r_A \omega$
- **Pin at $D$**

  **Velocity of Rubbing** $= r_D \omega_{CD}$
- **Pin at $B$**

  **Velocity of Rubbing** $= r_B(\omega_{AB} + \omega_{BC})$
- **Pin at $C$**

  **Velocity of Rubbing** $= r_C(\omega_{BC} + \omega_{CD})$

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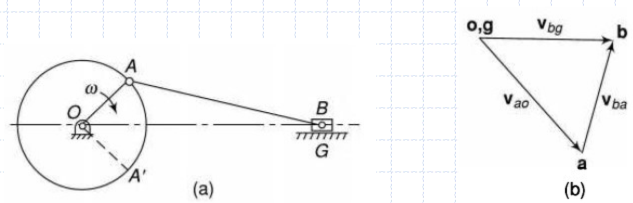
# Four Link Mechanism

- **Slider Crank Mechanism**

**OG: Fixed Link, OA: Crank, AB: Coupler, B: Slider**

$\omega$: *Angular Speed of OA (Clockwise)*




$\vec{V}_{bo} = \vec{V}_{ao} + \vec{V}_{ba}$ $\quad\quad$ $\vec{ob} = \vec{oa} + \vec{ab}$

**Where,** $\vec{V}_{ao}$ **or** $\vec{oa} = \omega OA$; $\perp$ **to** $\boldsymbol{O}A$

$\vec{V}_{ba}$ **or** $\vec{ab}$ **is unknown in magnitude;** $\perp$ **to** $AB$

$\vec{V}_{bo}$ **or** $\vec{ob}$ **is unknown in magnitude; ∥ to the motion of *B***

**Angular Velocity of Link AB:** $\omega_{AB} = \frac{v_{ba}}{AB}$ **( C.C.W )**

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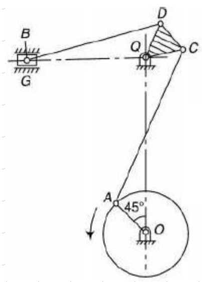
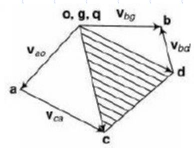
## Problem 1.




| | |
|---|---|
| $OA = 150\ mm$ | $CD = 125\ mm$ |
| $AC = 600\ mm$ | $BD = 500\ mm$ |
| $CQ = QD = 145\ mm$ | $OQ = 625\ mm$ |

$N_{OA} = 60rpm$ **(c.c.w)**

$\vec{V}_{bo} = ?$, $\omega_{BD} = ?$

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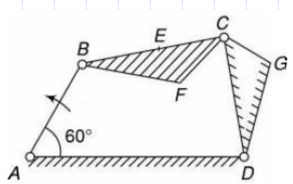
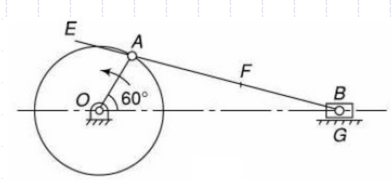
## Problem 2.




## Problem 3.

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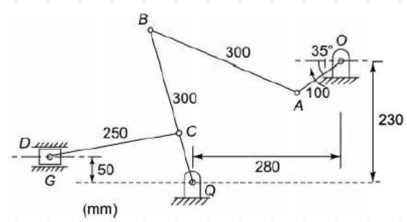
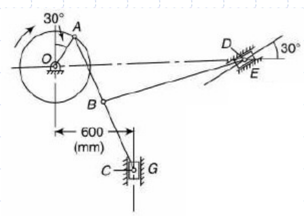
**Problem 4.**




**Problem 5.**

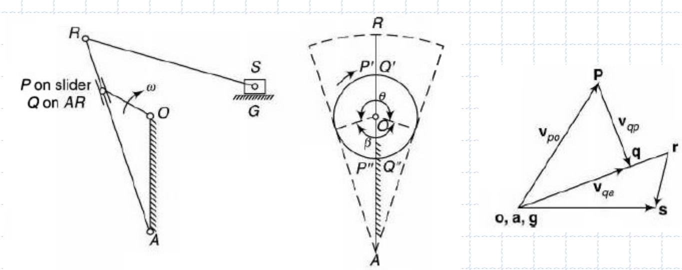





# Crank and Slotted Lever Mechanism

**OA: Fixed Link, OP: Crank, P, S: Slider, AR: Oscillating Link**

$\omega$: ***Angular Speed of OP (Clockwise)***



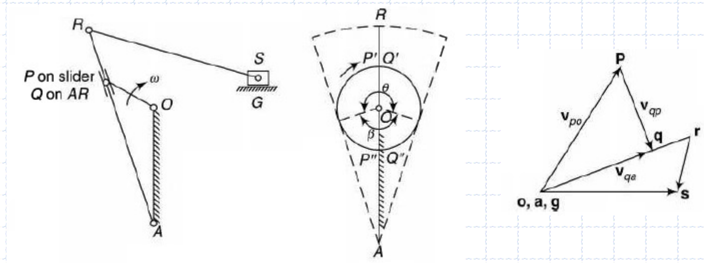

$\vec{V}_{qo}=\vec{V}_{po}+\vec{V}_{qp}$ or $\vec{V}_{qa}=\vec{V}_{po}+\vec{V}_{qp}$ or $\overrightarrow{aq}=\overrightarrow{op}+\overrightarrow{pq}$

**Where,** $\vec{V}_{po}$ **or** $\overrightarrow{op} = \omega OP$; $\perp$ **to** $OP$

$\vec{V}_{qp}$ **or** $\overrightarrow{pq}$ **is unknown in magnitude;** $\parallel$ **to** $AR$

$\vec{V}_{qa}$ **or** $\overrightarrow{aq}$ **is unknown in magnitude;** $\perp$ **to** $AR$

# Crank and Slotted Lever Mechanism




$\vec{V}_{ra}$ ⊥ to $AR$

$r$ **will lie on $\overrightarrow{aq}$ Produced such that $ar/aq = AR/AQ$**

$\vec{V}_{sa}=\vec{V}_{ra}+\vec{V}_{sr}$ $\quad$ $\overrightarrow{as}=\overrightarrow{ar}+\overrightarrow{rs}$

$\vec{V}_{sr}$ **or $\overrightarrow{rs}$ is unknown in magnitude; ⊥to $RS$**

$\vec{V}_{sa}$ **or $\overrightarrow{as}$ is unknown in magnitude; ∥ to the motion of *S***

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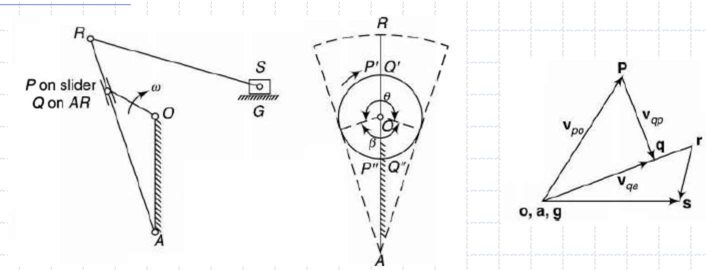
# Crank and Slotted Lever Mechanism




$$\frac{Time\ of\ cutting}{Time\ of\ Return} = \frac{\theta}{\beta}$$

At $P'$, during cutting stroke, $pq=0$ and $oq$ is maximum= $op$

Let

$r$ = length of crank (= $OP$)

$l$ = length of slotted lever (= $AR$)

$c$ = distance between fixed centres (= $AO$)

$\omega$ = angular velocity of the crank

during the cutting stroke,

$$v_{s\,max} = \omega \times OP' \times \frac{AR}{AQ} = \omega r \times \frac{l}{c+r}$$

during the return stroke,

$$v_{s\,max} = \omega \times OP'' \times \frac{AR}{AQ''} = \omega r \times \frac{l}{c-r}$$

$$\frac{v_{s\,max}\,(\text{cutting})}{v_{s\,max}\,(\text{return})} = \frac{\omega r \dfrac{1}{c+r}}{\omega r \dfrac{1}{c-r}} = \frac{c-r}{c+r}$$

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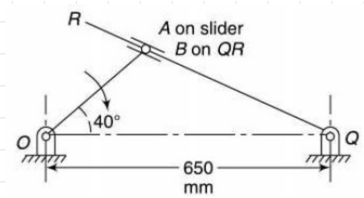
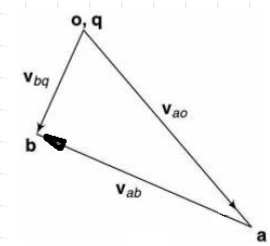
# Crank and Slotted Lever Mechanism

**Problem 1.**




$\vec{V}_{bq}=\vec{V}_{ao}+\vec{V}_{ba}$ $\quad \overrightarrow{qb}=\overrightarrow{oa}+\overrightarrow{ab}$

**Where,** $\vec{V}_{ao}$ **or** $\overrightarrow{oa} = \omega OA$; $\perp$ **to** $\boldsymbol{O}A$

$\vec{V}_{ba}$ **or** $\overrightarrow{ab}$ **is unknown in magnitude;** $\parallel$ **to** $QR$

$\vec{V}_{bq}$ **or** $\overrightarrow{qb}$ **is unknown in magnitude;** $\perp$ **to** $QR$

$$\omega_{qr} = \omega_{qb} = \frac{v_{qb} \text{ or } v_{bq}}{BQ}$$

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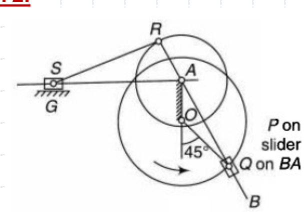
# Crank and Slotted Lever Mechanism

**Problem 2.**




$OP = 240\ mm, OA = 150\ mm, AR = 165\ mm$

$RS = 430\ mm, \omega_{op} = 2.5\ rad/s$

**(1) velocity of the ram $S$.**

**(2) velocity of the slider $P$ on the slotted lever.**

**(3) angular velocity of the link $RS$**

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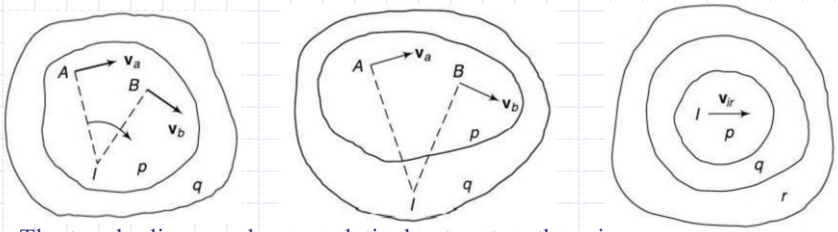
## Instantaneous Centre

- If the displacement of a rigid body having plane motion is considered as equivalent to a pure rotation of the body as a whole about some centre, such centre is called instantaneous centre.
- Any point on a rigid body or on its extension that has zero velocity is called the Instantaneous Center of Velocity of the body.




- The two bodies p and q are relatively at rest or there is no relative motion between the two at the I-centre.
- If body q is also in motion relative to a third body r, then the motion of the point I relative to the third body would be the same whether this point is considered on the body p or q.

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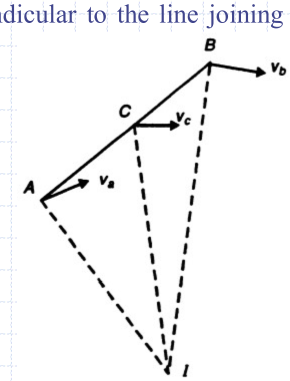
## Instantaneous Centre

- Velocity of any point in the body is proportional to its distance from the Instantaneous centre
- The direction of velocity is in a line perpendicular to the line joining that point to the Instantaneous centre.

$$\frac{v_c}{IC} = \frac{v_a}{IA} = \frac{v_b}{IB} = \text{angular velocity of link } AB$$




- Thus an Instantaneous centre makes it possible to determine the velocity of any point on a link if velocities of at least two points on the link or location of its instantaneous centre are known.

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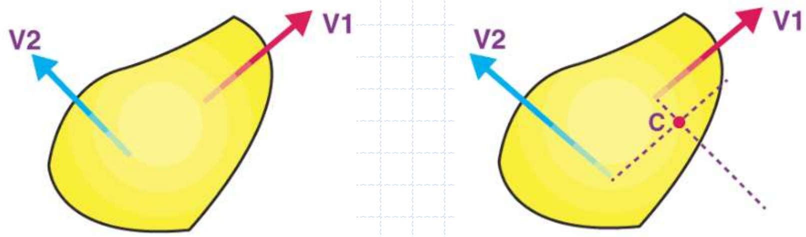
## Instantaneous Centre

**How can we locate the instantaneous centre of rotation?**

Case I : When two arbitrary (Non Parallel) velocities are given:




- Draw a perpendicular from the tails of two velocity vectors, the point where they intersect is the ICOR.

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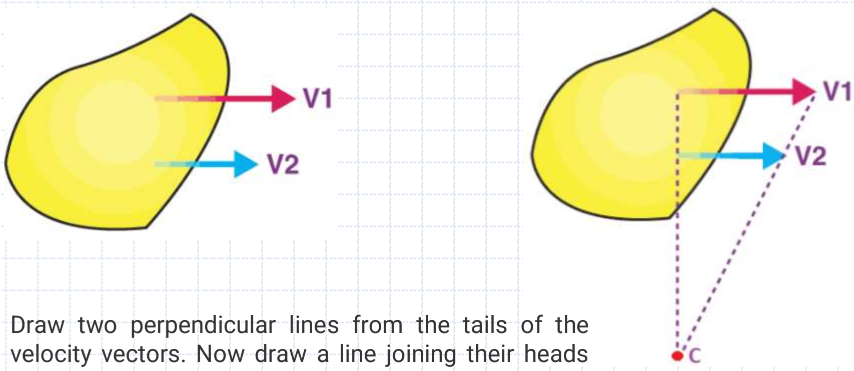
## Instantaneous Centre

**How can we locate the instantaneous centre of rotation?**

Case II: When two Parallel velocities are given in the same direction as shown:




- Draw two perpendicular lines from the tails of the velocity vectors. Now draw a line joining their heads which intersects the perpendicular line. That point of intersection is the ICOR.

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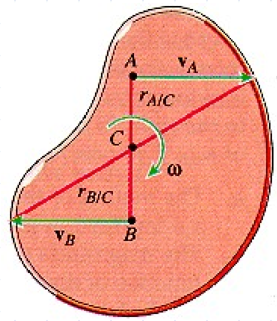
## Instantaneous Centre

**How can we locate the instantaneous centre of rotation?**

Case III: When two Parallel velocities are given in the opposite direction as shown:




- Draw two perpendicular lines from the tails of the velocity vectors. Now draw a line joining their heads which intersects the perpendicular line. That point of intersection is the ICOR.

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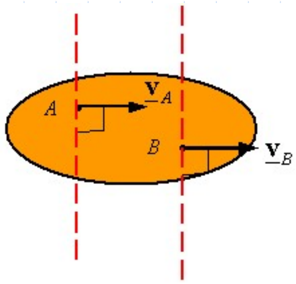
## Instantaneous Centre

**How can we locate the instantaneous centre of rotation?**

Case IV: When two Parallel and equal velocities are given in same direction as shown:

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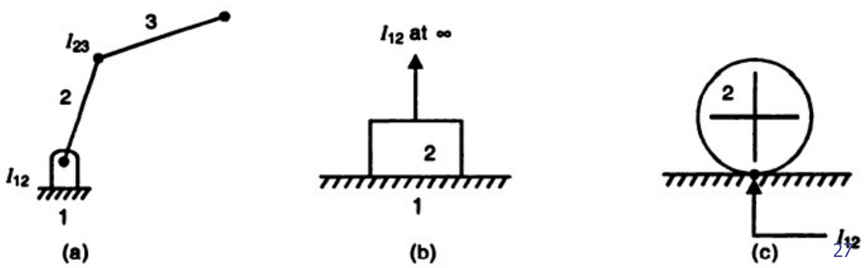
## Instantaneous Centre

- Instant center of velocities is a simple graphical method for performing velocity analysis on mechanisms.
- The method provides visual understanding on how velocity vectors are related.
- The location of instantaneous centres for various types of motion in a mechanism can be decided on the basis of the following rules:
  1. When two links form a turning pair, the instantaneous centre is assumed to be located at the centre of the pair [Figure (a)].
  2. In case of sliding pair, the instantaneous centre lies at infinity in the direction perpendicular to the path of motion of the slider [Figure (b)].
  3. When two links make a pure rolling contact, the point of contact at a given instant is taken as instantaneous centre [Figure (c)].

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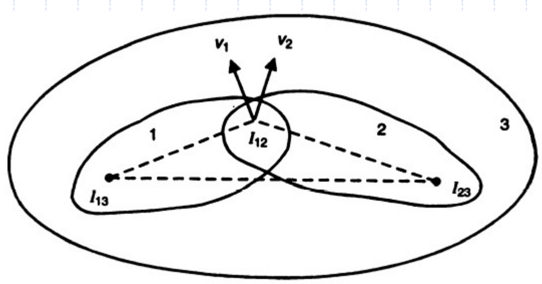
## Arnold Kennedy Theorem

- If three links have relative motion with respect to each other, their relative instantaneous centre lies on a straight line.

or

- The three instant centers between three planar links must lie on a straight line.

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## Locating Instantaneous Centres

Determine the Number of Instantaneous Centres

In a mechanism with n links (count the ground as one of the links), the number of instant centers is determined as:

$$C = n(n-1)/2$$

e.g.: In a four-bar mechanism or a slider-crank, there are six IC's ( n = 4 ).

For any six-bar mechanism, C = 15 .

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## Locating Instantaneous Centres

Instant Center Between a Link and The Ground

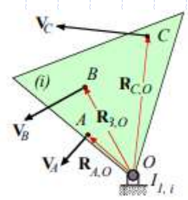
a) link i that is pinned to the ground at O

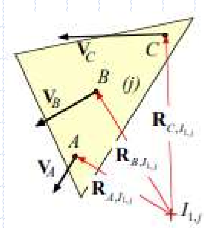
The velocity of any point on the link is determined as

$$\mathbf{V} = \omega \mathbf{R}$$

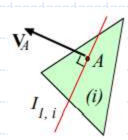
b) link j that is not connected to the ground directly

$$V_A / R_{A,I_{1,j}} = V_B / R_{B,I_{1,j}} = V_C / R_{C,I_{1,j}} = \omega$$




**Note:** If we know the velocity (absolute) of a point on a link, the instant center between that link and the ground must be located on an axis perpendicular to the velocity vector passing through the point.

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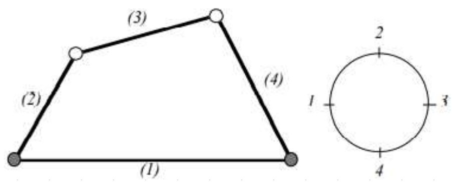
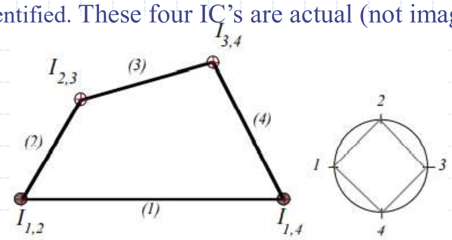
## Locating Instantaneous Centres

### Instant Centers of A Four-bar Mechanism

A four-bar mechanism has six instant centers regardless of the dimensions or orientation of the links.




Pin joints are instant centers, for a four bar with four pin joints, four IC's are immediately identified. These four IC's are actual (not imaginary) pin joints.





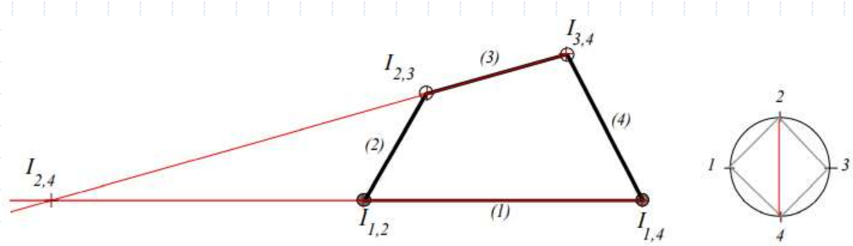

## Locating Instantaneous Centres

### Instant Centers of A Four-bar Mechanism

In order to find the other IC's, we apply Kennedy's rule.

The IC's between links 2, 3 and 4 must lie on a straight line. These are $I_{23}$ , $I_{34}$ , and $I_{24}$ .

The IC's between links 1, 2 and 4 must lie on a straight line. These are $I_{12}$ , $I_{14}$ , and $I_{24}$ .

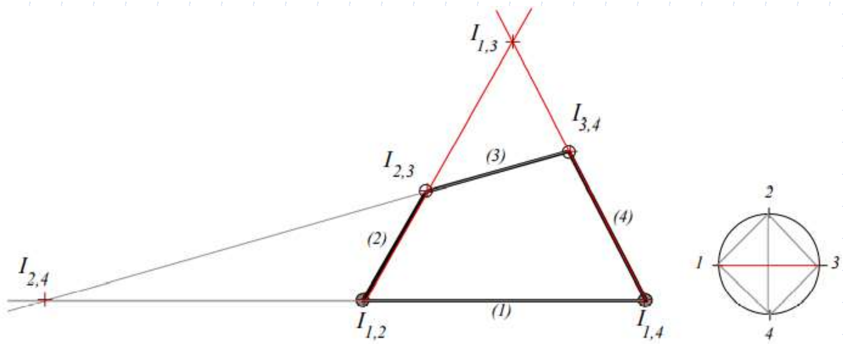


## Locating Instantaneous Centres

**Instant Centers of A Four-bar Mechanism**

In order to find the other IC's, we apply Kennedy's rule.

The last center to find is between links 1 and 3. The IC's between links 1, 2 and 3 must lie on a straight line. These are $I_{12}$ , $I_{23}$ , and $I_{13}$ .
The IC's between links 1, 4 and 3 must lie on a straight line. These are $I_{14}$ , $I_{34}$ , and $I_{13}$ .

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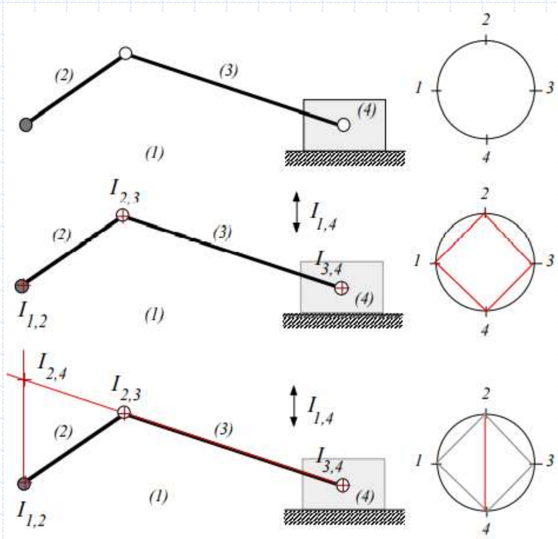
## Locating Instantaneous Centres

**Instantaneous Centers of A Slider Crank Mechanism Mechanism**

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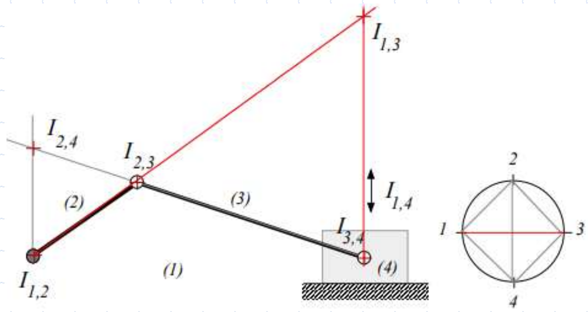
## Locating Instantaneous Centres

**Instantaneous Centers of A Slider Crank Mechanism Mechanism**

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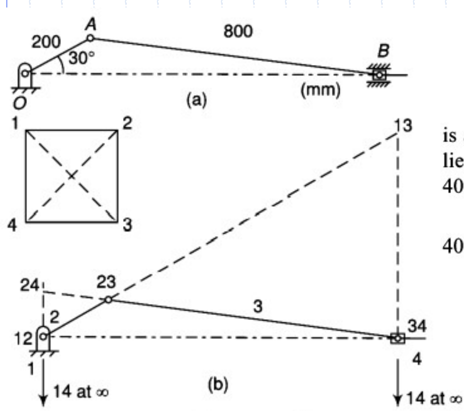


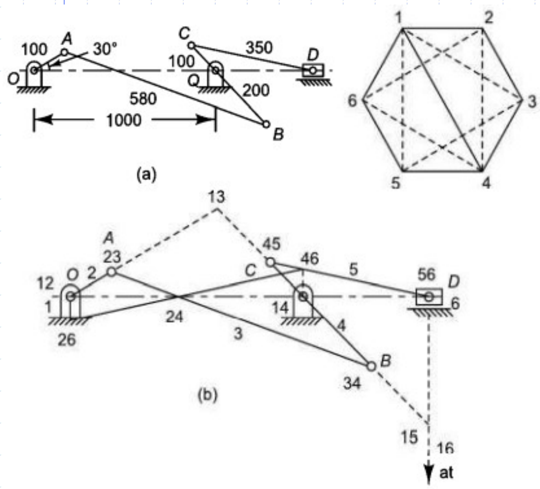

The point 24 has the same velocity whether it is assumed to lie in link 2 or 4. First, assume 24 to lie on the link 2 which rotates at angular velocity of 40 rad/s.

Linear velocity of I-centre 24 = 40 × (12–24) = 40 × 0.123

= 4.92 m/s in the horizontal direction

Thus, linear velocity of the slider = 4.92 m/s

36




Now, at the instance, the link 3 rotates about the I-centre 13.

$$\frac{v_b}{v_a} = \frac{13-34}{13-23} \text{ or } v_b = \frac{453}{265} \times 1.57 = 2.66 \text{ m/s}$$

$$\frac{v_c}{v_b} = \frac{QC}{QB} \text{ or } v_c = \frac{100}{200} \times 2.66 = 1.33 \text{ m/s}$$

At the instance, the link 5 rotates about the I-centre 15.

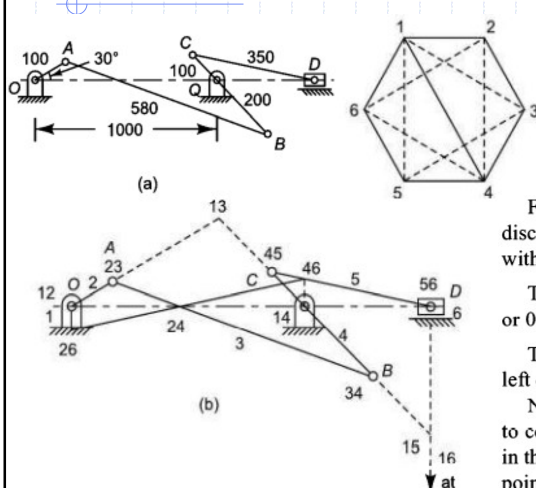
$$\text{Thus, } \frac{v_d}{v_c} = \frac{15-56}{15-45} \text{ or}$$

$$v_d = \frac{300}{506} \times 1.33 = 0.788 \text{ m/s}$$

$$\omega_{ab} = \frac{v_a}{13-A} = \frac{1.57}{0.267} = 5.88 \text{ rad/s}$$

$$\omega_{bc} = \frac{v_b}{BQ} = \frac{2.66}{0.2} = 13.3 \text{ rad/s}$$

$$\omega_{cd} = \frac{v_c}{15-c} = \frac{1.33}{0.499} = 2.66 \text{ rad/s}$$




First, imagine the link 2 to be in the form of a flat disc containing the point 26 and revolving about $O$ with an angular velocity of 15.7 rad/s.

Then, $v_{26} = \omega_2 \times (12\text{–}26) = 15.7 \times 50 = 785$ mm/s or 0.785 m/s

The velocity of the point 26 is in the horizontally left direction if $OA$ rotates clockwise.

Now, imagine the link 6 (slider) to be large enough to contain the point 26. The slider can have motion in the horizontal direction only and the velocity of a point 26 on it is known; it implies that all the points on the slider move with the same velocity.

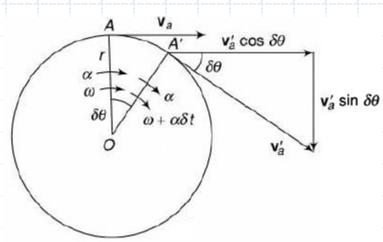
Thus, velocity of the slider, $v_d = v_{26} = \underline{785 \text{ mm/s}}$

38

# Acceleration analysis

# (Graphical Method)

## Acceleration




Angular velocity of $OA = \omega$

Tangential velocity of $A$, $v_a = \omega r$

Angular velocity of $OA'$, $\omega'_a = \omega + \alpha\,\delta t$

Tangential velocity of $A'$, $v'_a = (\omega + \alpha\,\delta t)\,r$

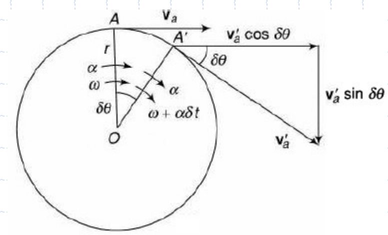
**Change of Velocity Perpendicular to $OA$**

$$\text{Acceleration of } A \perp \text{ to } OA = \frac{v'_a \cos \delta\theta - v_a}{\delta t}$$

$$\text{Acceleration of } A \perp \text{ to } OA = \frac{(\omega + \alpha.\delta t) r \cos \delta\theta - \omega r}{\delta t}$$

2

## Acceleration




Acceleration of $A \perp$ to $OA = \alpha r = \left(\frac{d\omega}{dt}\right) r = \frac{dv}{dt}$ ...$\left(\alpha = \frac{d\omega}{dt}\right)$ In the limit, as $\delta t \to 0$, $\sin \delta\theta \to \delta\theta$

This is known as Tangential acceleration of $A$ relative to $O$.

$$f_{ao}^t = \frac{dv}{dt}$$

**Change of Velocity Parallel to $OA$**

$$\text{Acceleration of } A \parallel \text{ to } OA = \frac{v'_a \sin \delta\theta - 0}{\delta t}$$

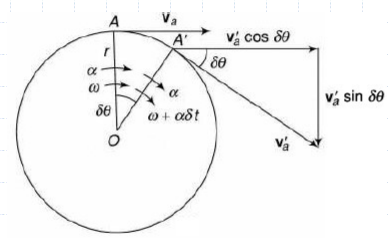
$$\text{Acceleration of } A \parallel \text{ to } OA = \frac{(\omega + \alpha\delta t) r \sin \delta\theta}{\delta t}$$

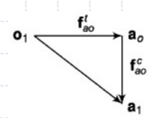
$$\text{Acceleration of } A \parallel \text{ to } OA = \omega^2 r = \frac{v^2}{r}$$ In the limit, as $\delta t \to 0$, $\sin \delta\theta \to \delta\theta$

This is known as Centripetal or Radial acceleration of $A$ relative to $O$.

3

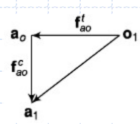
## Acceleration




$$f_{ao}^c = \omega^2 r = \frac{v^2}{r}$$

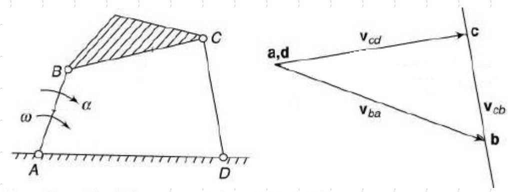
When $\alpha = 0$, $f_{ao}^t$=0 and $f_{ao}^c$ represents the total acceleration.
When $\omega = 0$, $f_{ao}^c$=0 and $f_{ao}^t$ represents the total acceleration.
When $\alpha$ is negative, $f_{ao}^t$ is negative.

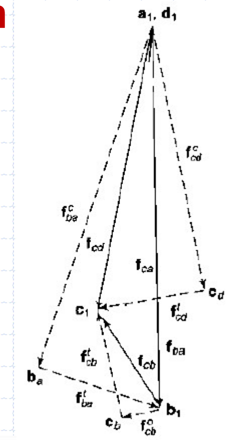



4

# Four Link Mechanism

## 4R Planar Mechanism




$\mathbf{f_{ca}} = \mathbf{f_{cd}} = \mathbf{f_{ba}} + \mathbf{f_{cb}}$ $\qquad$ $\mathbf{d_1c_1} = \mathbf{a_1b_1} + \mathbf{b_1c_1}$

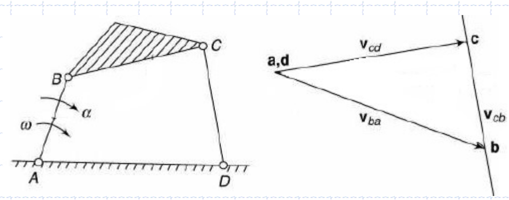
$\mathbf{f^c_{cd}} + \mathbf{f^t_{cd}} = \mathbf{f^c_{ba}} + \mathbf{f^t_{ba}} + \mathbf{f^c_{cb}} + \mathbf{f^t_{cb}}$

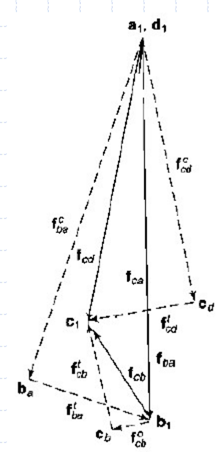
$\mathbf{d_1\,c_d} + \mathbf{c_d\,c_1} = \mathbf{a_1\,b_a} + \mathbf{b_a\,b_1} + \mathbf{b_1\,c_b} + \mathbf{c_b\,c_1}$

| SN | Vector | Magnitude | Direction | Sense |
|---|---|---|---|---|
| 1. | $f^c_{ba}$ or $\mathbf{a_1b_a}$ | $\frac{(\mathbf{ab})^2}{AB}$ | $\parallel AB$ | $\rightarrow A$ |
| 2. | $f^t_{ba}$ or $\mathbf{b_ab_1}$ | $\alpha \times AB$ | $\perp AB$ or $\mathbf{a_1\,b_a}$ or $\parallel \mathbf{ab}$ | $\rightarrow \mathbf{b}$ |
| 3. | $f^c_{cb}$ or $\mathbf{b_1c_b}$ | $\frac{(\mathbf{bc})^2}{BC}$ | $\parallel BC$ | $\rightarrow B$ |
| 4. | $f^t_{cb}$ or $\mathbf{c_bc_1}$ | - | $\perp BC$ or $\mathbf{b_1c_b}$ | - |
| 5. | $f^c_{cd}$ or $\mathbf{d_1c_d}$ | $\frac{(\mathbf{dc})^2}{DC}$ | $\parallel DC$ | $\rightarrow D$ |
| 6. | $f^t_{cd}$ or $\mathbf{c_dc_1}$ | - | $\perp DC$ or $\mathbf{d_1c_d}$ | - |


# Four Link Mechanism

## 4R Planar Mechanism


**Angular Acceleration of Links**

Link $BC$

$\mathbf{f^t_{cb}} = \mathbf{c_b\,c_1}$

$f^t_{cb} = \alpha_{cb} CB \quad \therefore \alpha_{cb} = f^t_{cb}/CB \quad (C.C.W)$

Link $CD$

$\mathbf{f^t_{cd}} = \mathbf{c_d\,c_1} \quad \therefore \alpha_{cd} = \frac{f^t_{cd}}{CD} = \frac{c_dc_1}{CD}$

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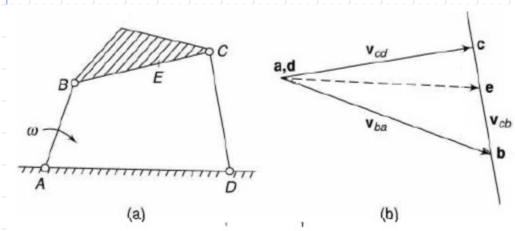
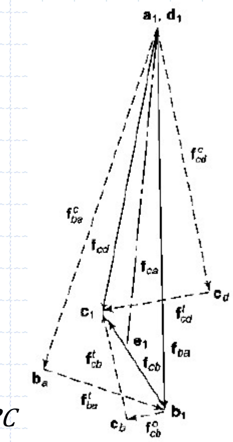
# Four Link Mechanism

- **4R Planar Mechanism**

**Acceleration of Intermediate Point (Point E on Link $BC$)**




$$\frac{b_1e_1}{b_1c_1} = \frac{BE}{BC}$$

**Here,** $\overrightarrow{a_1e_1}$: *Total Acceleration of Point E on Link BC*

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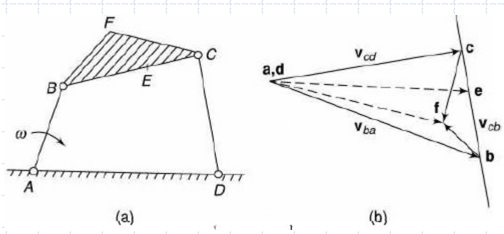
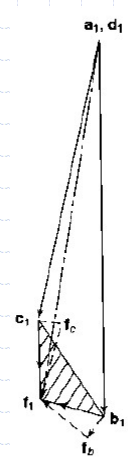
# Four Link Mechanism

- **4R Planar Mechanism**

**Acceleration of Offcet Point (Point F on Link $BC$)**




**$\Delta b_1c_1f_1$ is similar to $\Delta BCF$.**

**$\Delta b_1c_1f_1$ is acceleration image.**

**Here,** $\overrightarrow{a_1f_1}$: *Total Acceleration of Point F on Link BC*

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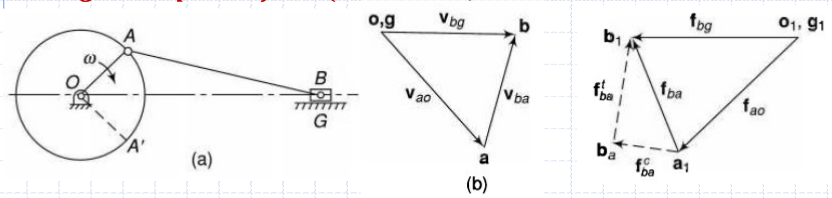
# Four Link Mechanism

◆ **3R-1P Planar Mechanism (Slider Crank Mechanism)**

**OG: Fixed Link, OA: Crank, AB: Coupler, B: Slider**

$\omega$: ***Angular Speed of OA (Clockwise)***




$$\mathbf{f}_{bo} = \mathbf{f}_{ba} + \mathbf{f}_{ao}$$
$$\mathbf{f}_{bg} = \mathbf{f}_{ao} + \mathbf{f}_{ba} = \mathbf{f}_{ao} + \mathbf{f}^{c}_{ba} + \mathbf{f}^{t}_{ba}$$
$$\mathbf{g_1\,b_1 = o_1\,a_1 + a_1\,b_a + b_a\,b_1}$$

| SN | Vector | Magnitude | Direction | Sense |
|---|---|---|---|---|
| 1. | $\mathbf{f}_{ao}$ or $\mathbf{o}_1\mathbf{a}_1$ | $\frac{(\mathbf{oa})^2}{OA}$ | $\parallel OA$ | $\rightarrow$ O |
| 2. | $\mathbf{f}^{c}_{ba}$ or $\mathbf{a}_1\,\mathbf{b}_a$ | $\frac{(\mathbf{ab})^2}{AB}$ | $\parallel AB$ | $\rightarrow$ A |
| 3. | $\mathbf{f}^{t}_{ba}$ or $\mathbf{b}_a\,\mathbf{b}_1$ | - | $\perp AB$ | - |
| 4. | $\mathbf{f}_{bg}$ or $\mathbf{g}_1\mathbf{b}_1$ | - | $\parallel$ to line of motion of $B$ | - |

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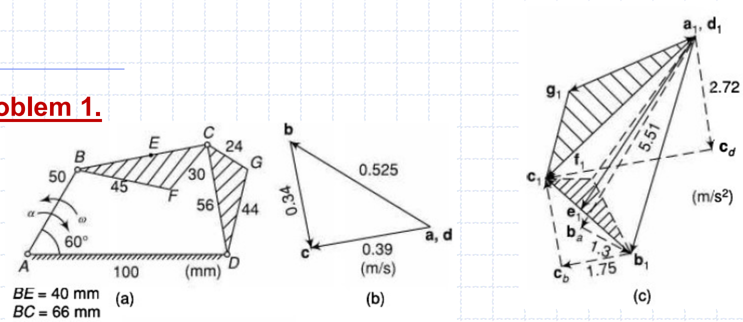
**Problem 1.**




| SN | Vector | Magnitude (m/s²) | Direction | Sense |
|---|---|---|---|---|
| 1. | $\mathbf{f}^{c}_{ba}$ or $\mathbf{a}_1\,\mathbf{b}_a$ | $\frac{(\mathbf{ab})^2}{AB} = \frac{(0.525)^2}{0.05} = 5.51$ | $\parallel AB$ | $\rightarrow A$ |
| 2. | $\mathbf{f}^{t}_{ba}$ or $\mathbf{b}_a\mathbf{b}_1$ | $\alpha \times AB = 26 \times 0.05 = 1.3$ | $\perp AB$ or $\parallel$ **ab** | $\rightarrow$ **a** |
| 3. | $\mathbf{f}^{t}_{cb}$ or $\mathbf{b}_1\mathbf{c}_b$ | $\frac{(\mathbf{bc})^2}{BC} = \frac{(0.34)^2}{0.066} = 1.75$ | $\parallel BC$ | $\rightarrow B$ |
| 4. | $\mathbf{f}^{t}_{cb}$ or $\mathbf{b}_b\mathbf{c}_1$ | - | $\perp B$ | - |
| 5. | $\mathbf{f}^{c}_{cd}$ or $\mathbf{d}_1\mathbf{c}_d$ | $\frac{(\mathbf{dc})^2}{DC} = \frac{(0.39)^2}{0.56} = 2.72$ | $\parallel DC$ | $\rightarrow D$ |
| 6. | $\mathbf{f}^{t}_{cd}$ or $\mathbf{c}_d\mathbf{c}_1$ | - | $\perp B$ | - |

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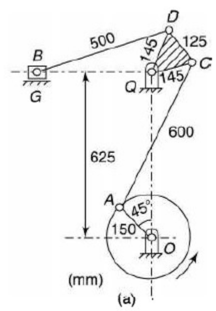
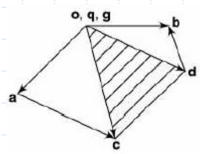
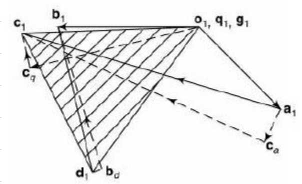
## Problem 2.



(a)



(b)



(c)

$\mathbf{f}_{cq} = \mathbf{f}_{ao} + \mathbf{f}_{ca}$

$\mathbf{q_1 c_1 = o_1 a_1 + a_1 c_1}$

$\mathbf{f}^t_{cq} + \mathbf{f}^t_{cq} = \mathbf{f}^t_{ao} + \mathbf{f}^t_{ca} + \mathbf{f}^t_{ca}$

$\mathbf{q_1 c_q + c_q c_1 = o_1 a_1 + a_1 c_a + c_a a_1}$

Draw $\Delta \mathbf{c_1 q_1 d_1}$ similar to $\Delta CQD$

$\mathbf{f}_{bg} = \mathbf{f}_{dq} + \mathbf{f}_{bd}$

$\mathbf{g_1 b_1 = q_1 d_1 + d_1 b_1}$

$\mathbf{f}_{bg} = \mathbf{f}_{dq} + \mathbf{f}^c_{bd} + \mathbf{f}^t_{bd}$

$\mathbf{g_1 b_1 = q_1 d_1 + d_1 b_d + b_d b_1}$

| SN | Vector | Magnitude (m/s²) | Direction | Sense |
|---|---|---|---|---|
| 1. | $\mathbf{f}_{ao}$ or $\mathbf{o_1 a_1}$ | $\frac{(\mathbf{oa})^2}{OA} = \frac{(0.94)^2}{0.15} = 5.92$ | $\parallel OA$ | $\to O$ |
| 2. | $\mathbf{f}^c_{ca}$ or $\mathbf{a_1 c_a}$ | $\frac{(\mathbf{ac})^2}{AC} = \frac{(1.035)^2}{0.60} = 1.79$ | $\parallel AC$ | $\to A$ |
| 3. | $\mathbf{f}^t_{ca}$ or $\mathbf{c_a c_1}$ | - | $\perp AC$ | - |
| 4. | $\mathbf{f}^c_{cq}$ or $\mathbf{q_1 c_q}$ | $\frac{(\mathbf{qc})^2}{QC} = \frac{(1.14)^2}{0.145} = 8.96$ | $\parallel QC$ | $\to Q$ |
| 5. | $\mathbf{f}^t_{cq}$ or $\mathbf{c_q c_1}$ | - | $\perp QC$ | - |

| SN | Vector | Magnitude (m/s²) | Direction | Sense |
|---|---|---|---|---|
| 1. | $\mathbf{f}_{dq}$ or $\mathbf{q_1 d_1}$ | Already drawn | - | - |
| 2. | $\mathbf{f}^c_{bd}$ or $\mathbf{d_1 b}_d$ | $\frac{(\mathbf{db})^2}{DB} = 0.49$ | $\parallel DB$ | $\to D$ |
| 3. | $\mathbf{f}^t_{bd}$ or $\mathbf{b}_d\ \mathbf{b}_1$ | - | $\perp DB$ | - |
| 4. | $\mathbf{f}_{bg}$ or $\mathbf{g_1 b_1}$ | - | $\parallel$ to slider motion | - |

$\alpha_{cqd} = \frac{\mathbf{f}^t_{cq} \text{ or } \mathbf{c}_q\mathbf{c}_1}{QC}$ $\qquad$ $\alpha_{bd} = \frac{\mathbf{f}^t_{bd} \text{ or } \mathbf{b}_d\mathbf{b}_1}{BD}$

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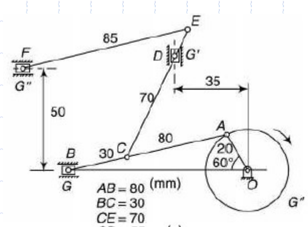
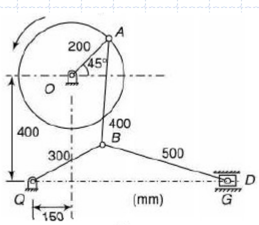
## Problem 3.



(a)

## Problem 4.



(a)

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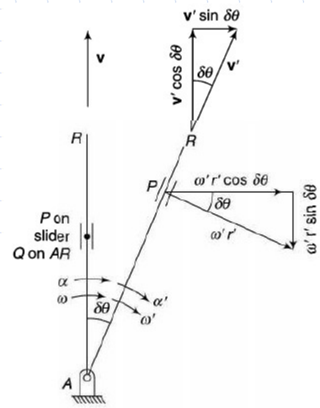
# Crank and Slotted Lever Mechanism

## Coriolis Acceleration Component

$\omega$ = angular velocity of the link
$\alpha$ = angular acceleration of the link
$v$ = linear velocity of the slider on the link
$f$ = linear acceleration of the slider on the link
$r$ = radial distance of point $P$ on the slider

$\omega' = \omega + \alpha\delta t$ = angular velocity of the link
$v' = v + f.\delta t$ = linear velocity of the slider on the link
$r' = r + \delta r$ = radial distance of the slider




**Acceleration of P Parallel to AR**

Initial velocity of $P$ along $AR = v = v_{pq}$
Final velocity of $P$ along $AR = v' \cos\delta\theta - \omega' r' \sin\delta\theta$
Change of velocity along $AR = (v' \cos\delta\theta - \omega' r' \sin\delta\theta) - v$
Acceleration of $P$ along $AR$

$$= \frac{(v + f\delta t)\cos\delta\theta - (\omega + \alpha\delta t)(r + \delta r)\sin\delta\theta - v}{\delta t}$$

In the limit, as $\delta t \to 0$
$\cos\delta\theta \to 1$ and $\sin\delta\theta \to \delta\theta$

$$\text{Acceleration of } P \text{ along } AR = f - \omega r \frac{d\theta}{dt} = f - \omega r w = f - \omega^2 r = \text{Acc. of slider–centripetal. acc.}$$

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# Crank and Slotted Lever Mechanism

## Coriolis Acceleration Component

## Acceleration of P Perpendicular to AR

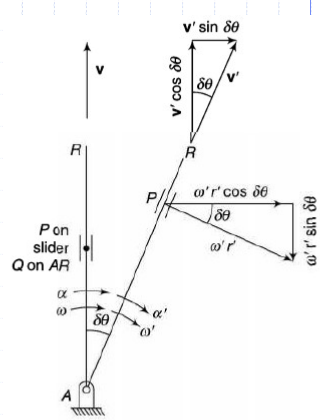
Initial velocity of $P \perp$ to $AR = \omega r$
Final velocity of $P \perp$ to $AR = v' \sin\delta\theta + \omega' r' \cos\delta\theta$
Change of velocity $\perp$ to $AR = (v' \sin\delta\theta + \omega' r' \cos\delta\theta) - \omega r$
Acceleration of $P \perp$ to $AR$

$$= \frac{(v + f\delta t)\cos\delta\theta - (\omega + \alpha\delta t)(r + \delta r)\cos\delta\theta - \omega r}{\delta t}$$

In the limit, as $\delta t \to 0$

$\cos\delta\theta \to 1$ and $\sin\delta\theta \to \delta\theta$

$$\text{Acceleration of } P \perp \text{ to } AR = v\frac{d\theta}{dt} + \omega\frac{dr}{dt} + ra = v\omega + \omega v + r\alpha = 2\omega v + r\alpha = 2\omega v + \text{tangential acc.}$$




The component $2\omega.v$ is known as the *Coriolis acceleration*

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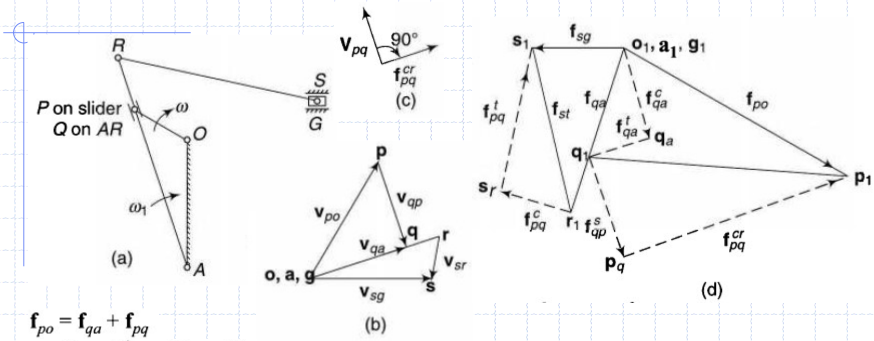
# Crank and Slotted Lever Mechanism




$$\mathbf{f}_{po} = \mathbf{f}_{qa} + \mathbf{f}_{pq}$$
$$= f^c_{qa} + f^t_{qa} + f^s_{pq} + f^{cr}_{pq}$$

$$\mathbf{o_1\, p_1 = a_1\, q_a + q_a\, q_1 + q_1\, p_q + p_q\, p_1}$$

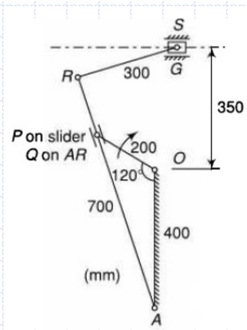
| SN | Vector | Magnitude | Direction | Sense |
|---|---|---|---|---|
| 1. | $\mathbf{f}_{po}$ or $\mathbf{o_1\, p_1}$ | $\omega^2 \times OP$ | $\parallel OP$ | $\rightarrow O$ |
| 2. | $\mathbf{f}^c_{qa}$ or $\mathbf{a}_1\, \mathbf{q}_a$ | $\dfrac{(\mathbf{aq})^2}{AQ}$ | $\parallel AQ$ | $\rightarrow A$ |
| 3. | $\mathbf{f}^t_{qa}$ or $\mathbf{q}_a\, \mathbf{q}_1$ | - | $\perp AQ$ or $\mathbf{a_1\, q_a}$ | - |
| 4. | $\mathbf{f}^s_{pq}$ or $\mathbf{q}_1\, \mathbf{p}_q$ | - | $\parallel AR$ | - |
| 5. | $\mathbf{f}^{cr}_{pq}$ or $\mathbf{p}_q\, \mathbf{p}_1$ | Coriolis component* | $\perp AR$ | Refer* |

$$* \; f^{cr}_{pq} = 2\omega_1 v_{pq}$$
$$= 2\left(\frac{\mathbf{aq}}{AQ}\right) qp$$



# Crank and Slotted Lever Mechanism

**Problem 1.**




**Problem 2.**