MEC2120
Kinematics of Machines

Syllabus

Unit – I : Kinematic pairs & chain, constrained criterion, mobility and range of movement, Planar Mechanisms and its inversion, Straight line motion Mechanisms, Pantograph, Engine indicator, Hook's joint and steering gear mechanism.

Unit–II : Velocity analysis in mechanism: relative velocity & Instantaneous centre method, Acceleration analysis in mechanism, Graphical method, problem involving Corriolis acceleration, Klien's construction, Analytical methods for velocity & acceleration analysis.

Unit–III : Kinematic Synthesis of Planar Mechanisms: Chebyshev Spacing of Precision Points, Two-/Three- position synthesis of Planar four bar mechanisms, Path Generation and Function generation problems, Bloch's Method and Freudenstein's method of synthesis.

Unit–IV : Gear Drives: Introduction, classification of gear, gear nomenclature, tooth profile, interference, path of contact, arc of contact of meshing gears. Gear Train: Simple, compound and epi-cyclic gear trains.

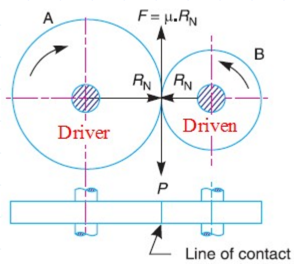
Books:

1. S S Ratan: Theory of Machines; McGraw Hill.
2. J S Rao: Mechanism & Machine Theory, New Age International.
3. Chales E Wilson & J Peter Sadler: Kinematics & Dynamics of Machinery; Pearson Education.

Unit 4

Gears and Gear Drives

Friction wheel



- *For no Slipping condition, the following kinematic relationship exists.*

$$v_p = \omega_A r_A = \omega_B r_B$$
$$\Rightarrow \quad \frac{\omega_A}{\omega_B} = \frac{r_B}{r_A}$$
$$\frac{N_A}{N_B} = \frac{r_B}{r_A}$$

- The friction drive is not a positive drive.
- The friction wheels can only be used for transmission of small powers.

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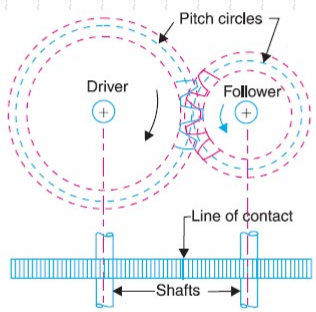
Toothed wheel/Gear



- A friction wheel with the teeth cut on it is known as ***toothed wheel* or *gear*.**

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Toothed wheel/Gear

Advantages

1. It transmits exact velocity ratio.
2. It may be used to transmit large power.
3. It has high efficiency.
4. It has reliable service.
5. It has compact layout.

Disadvantages

1. The manufacture of gears require special tools and equipment.
2. The error in cutting teeth may cause vibrations and noise during operation.

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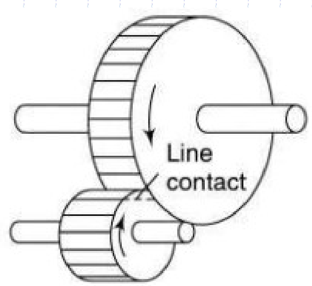
Classification of Toothed wheel/Gear

1. ***According to the position of axes of the shafts.***
   (*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

(*a*) Parallel,

spur gears



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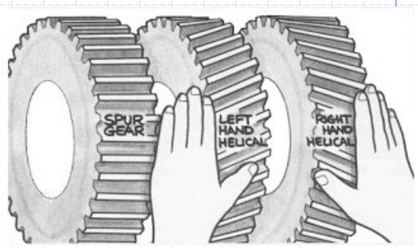
Classification of Toothed wheel/Gear

1. According to the position of axes of the shafts.

(*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

(*a*) Parallel,

helical gearing



- Mating helical gears (on parallel shafts) must have the same helix angle but the opposite hand.

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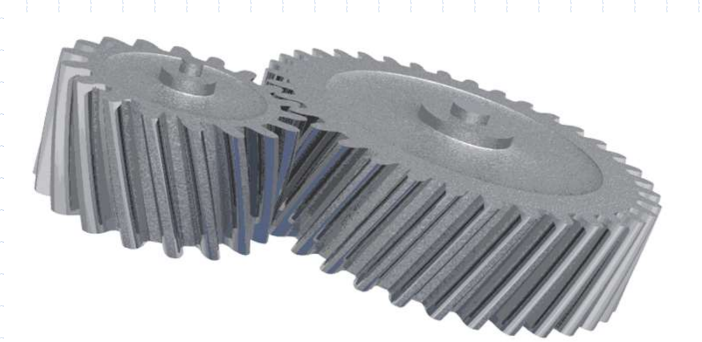
(*a*) Parallel,

helical gearing



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Classification of Toothed wheel/Gear

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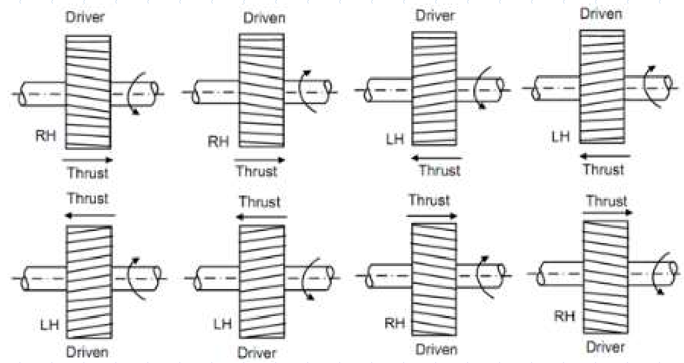

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Classification of Toothed wheel/Gear

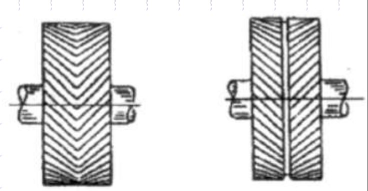
1. According to the position of axes of the shafts.

(*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

(*a*) Parallel,

herringbone gears

- To avoid axial thrust, two helical gears of opposite hand can be mounted side by side, to cancel resulting thrust forces.
- Herringbone gears are mostly used on heavy machinery.



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Classification of Toothed wheel/Gear

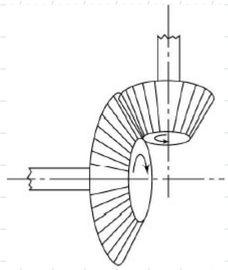
1. ***According to the position of axes of the shafts.***

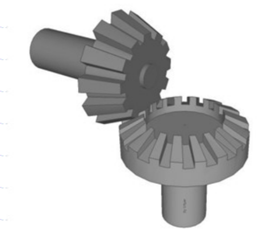
   (*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

(*a*) Intersecting

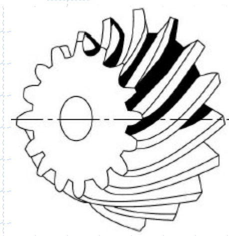
Bevel gears

Straight Bevel gears





Spiral or Helical Bevel gears



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Classification of Toothed wheel/Gear

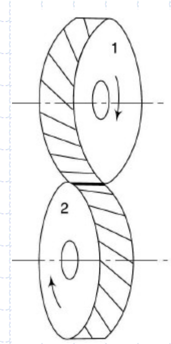
1. ***According to the position of axes of the shafts.***

   (*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

(*a*) Non-intersecting and non-parallel

Crossed Helical gears

- **These are used for light loads.**
- **By a suitable choice of helix angle for the mating gears, the two shafts can be set at any angle**
- **These gears theoretically give point contact, unlike spur or helical gears.**
- **Used for feed mechanism on machine tools, camshafts and oil pumps on small IC engines.**



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Classification of Toothed wheel/Gear

1. ***According to the position of axes of the shafts.***

   (*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

(*a*) Non-intersecting and non-parallel

Worm gears

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Classification of Toothed wheel/Gear

1. ***According to the position of axes of the shafts.***

   (*a*) Parallel, **(*b*)** Intersecting, and **(*c*)** Non-intersecting and non-parallel.

(*a*) Non-intersecting and non-parallel

Worm gears

- **Used to connect skew shafts (generally at right angles).**
- **Transmit high velocity ratios**
- **The worm gears give line contact between mating teeth unlike a point contact in the case of spiral gears (crossed helical gears).**
- **The load carrying capacity is improved by using enveloping worm, instead of a straight worm.**

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Classification of Toothed wheel/Gear

1. According to the position of axes of the shafts.

(***a***) Parallel, (***b***) Intersecting, and (***c***) Non-intersecting and non-parallel.

(*a*) Non-intersecting and non-parallel

Worm gear set types

1. Non-throated: The contact between the teeth is concentrated at a point.
2. Single-throated: Gear teeth are cuved to envelop the worm. There is a line contact between the teeth.
3. Double throated: There is area contact between the teeth.

Classification of Toothed wheel/Gear

2. According to the peripheral velocity of the gears.

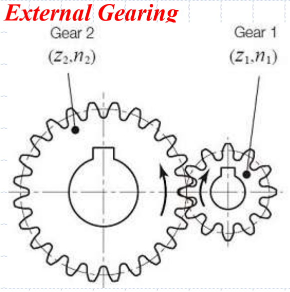
(***a***) Low velocity: Velocity is less than 3 m/sec,

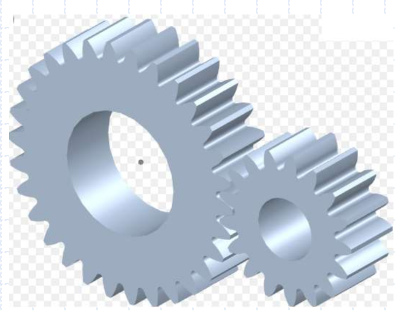
(***b***) Medium velocity: Velocity is between 3 m/sec to 15 m/sec, and

(***c***) High Velocity: Velocity is above 15 m/sec.

3. According to the type of gearing.

(a) External Gearing



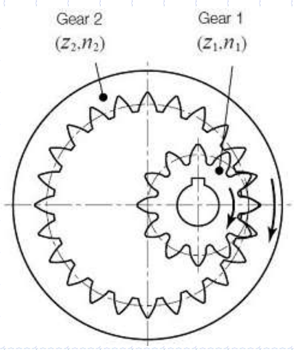


- External gears have the gear teeth generated on the outside diameter of the component.
- In the case of external toothing, the teeth are directed outwards on the circumference.

Classification of Toothed wheel/Gear

3. According to the type of gearing.

(b) Internal Gearing



- Internal Gears are **gear teeth generated in the internal diameter of a cylinder**.
- In the **case of internal gears, the teeth are directed inwards**.

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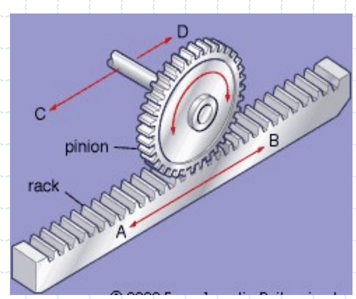
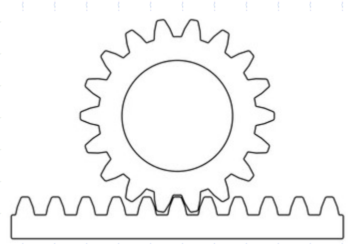
Classification of Toothed wheel/Gear

3. According to the type of gearing.

(c) Rack and Pinion

- A bar of rectangular cross section (the rack), having teeth on one side that mesh with teeth on a small gear (the pinion).
- If the pinion rotates about a fixed axis, the rack will translate; *i.e.,* move on a straight path. e.g. Some automobiles have rack-and-pinion drives on their steering mechanisms that operate in this way.
- If the rack is fixed and the pinion is carried in bearings on a table guided on tracks parallel to the rack, rotation of the pinion shaft will move the table parallel to the rack. e.g. On machine tools, rack-and-pinion mechanisms are used in this way to obtain rapid movements of worktables





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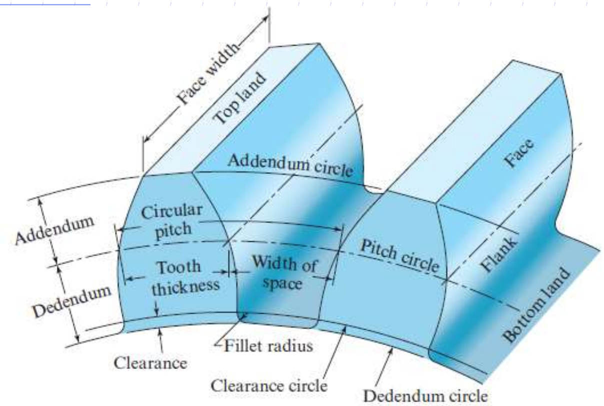
Gears



1. *Pitch circle.*
2. *Pitch circle diameter.*
3. *Pitch point.*
4. *Pitch surface.*
5. *Addendum circle.*
6. *Dedendum circle.*
7. *Addendum.*
8. *Dedendum.*
9. *Total depth*
10. *Clearence*
11. *Clearence circle*
12. *Top land.*
13. *Bottom land*
14. *Face*
15. *Flank*
16. *Tooth profile*
17. *Face Width*
18. *Tooth thickness*
19. *Tooth space*
20. *Circular pitch,* $P_c = \pi D/T$

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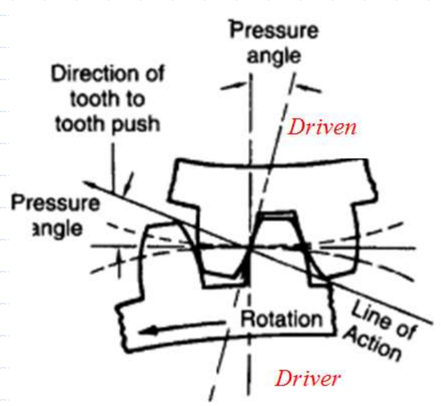
Gears

21. *Diametral Pitch,* $P_d = T/D = \pi/P_c$
22. *Module,* $m = D/T$
23. *Line of Action*
24. *Pressure angle*
25. *Path of contact*
26. *Arc of contact*



(Wheel 2)

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Law of Gearing

Q: Point of contact of two teeth.
T T : common tangent and
MN :common normal to the curves at the point of contact *Q*.
v_1 and v_2 : Velocities of the point *Q* on the wheels 1 and 2 respectively.
For the teeth to remain in contact, we must have

$$v_1 \cos\alpha = v_2 \cos\beta$$

$$(\omega_1 \times O_1 Q)\cos\alpha = (\omega_2 \times O_2 Q)\cos\beta$$

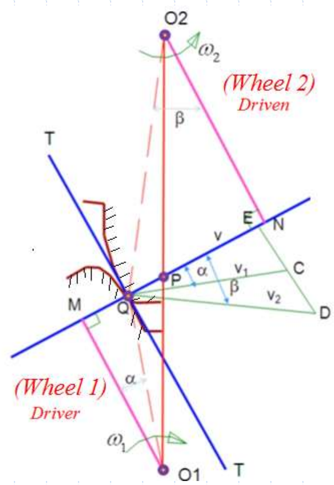
$$(\omega_1 \times O_1 Q)\frac{O_1 M}{O_1 Q} = (\omega_2 \times O_2 Q)\frac{O_2 N}{O_2 Q}$$

$$\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} \qquad Eq.(1)$$

Also we have

$$\frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P} \qquad (\because \Delta O_1MP \sim \Delta O_2NP)$$

$$Eq.(2)$$



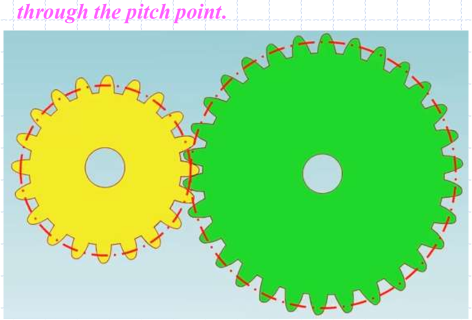
Law of Gearing

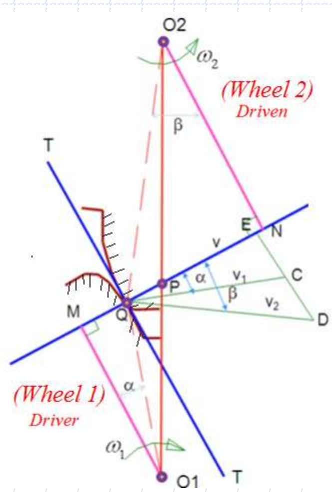
Combining Eq.1 and Eq.2

$$\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P}$$

∴ For constant angular velocity ratio for all positions of the wheels, the
point *P* must be the fixed point (called pitch point) for the two wheels.

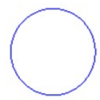
Law of Gearing: ***The common normal at the point of contact between a pair of teeth must always pass through the pitch point.***





Teeth Profiles satisfying the law of gearing

1) *Involute Profile*
2) *Cycloidal Profile*



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Teeth Profiles satisfying the law of gearing

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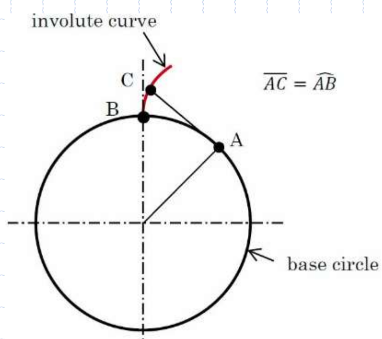

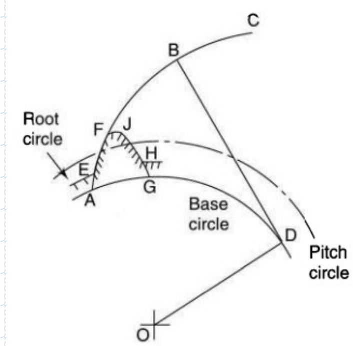

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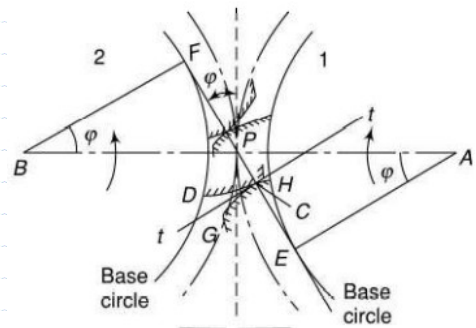
Teeth Profiles satisfying the law of gearing

1) Involute Profile

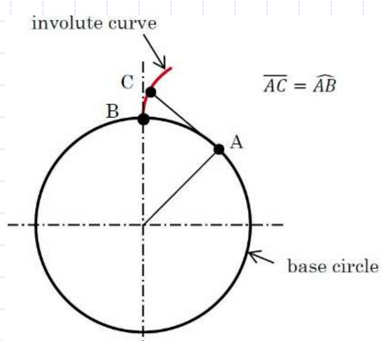


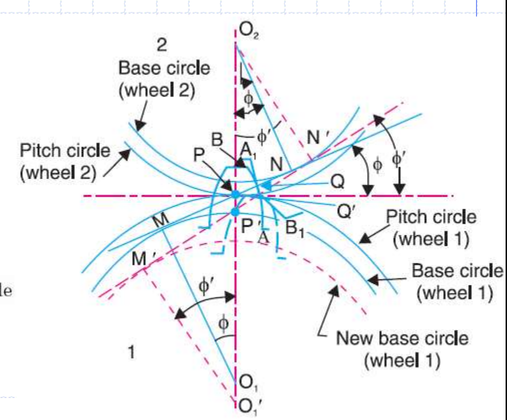
velocity ratio of gears $= \frac{BP}{AP} = \frac{BF}{AE} =$ constant

Thus, for a pair of involute gears, the velocity ratio is inversely proportional to the pitch circle diameters as well as base circle diameters.

Teeth Profiles satisfying the law of gearing

1) Involute Profile

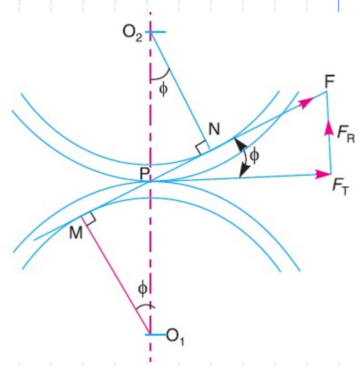




Teeth Profiles satisfying the law of gearing

Properties of Involute Profile

- The shape of involute profile is dependent only on the dimensions of the base circle.
- Angular velocity ratio of involute profile teeth is independent to centre distance of the base circles.
- if the centre distance is changed within limits, the velocity ratio remains unchanged. However, the pressure angle increases with the increase in the centre distance.
- When two involutes are in mesh, the angular velocity ratio is inversely proportional to the size of the base circles.
- The pressure angle of two involutes in mesh is constant.



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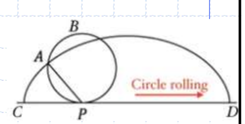
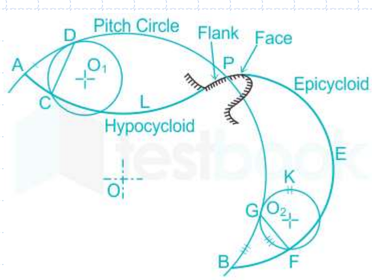
Teeth Profiles satisfying the law of gearing

Cycloidal Profile



The cycloid is defined as the locus of a point on the circumference of a circle that rolls without slipping on a fixed straight line.
In Cycloidal tooth profile, the faces of the teeth are epicycloids and the flanks are the hypocycloids.
An epicycloid is the locus of a point on the circumference of a circle that rolls without slipping on the circumference of another circle.
A hypocycloid is the locus of a point on the circumference of a circle that rolls without slipping inside the circumference of another circle.
Formation of cycloidal tooth :



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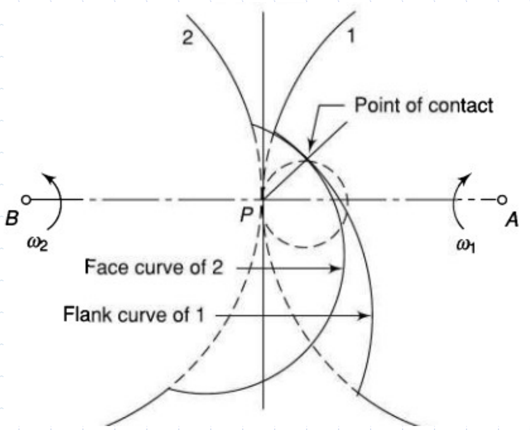
Teeth Profiles satisfying the law of gearing

Cycloidal Profile



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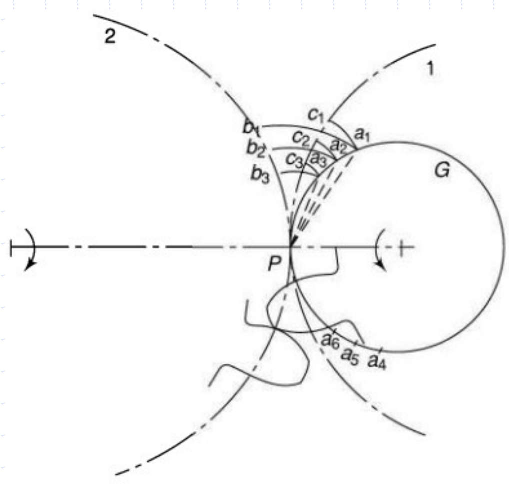
Teeth Profiles satisfying the law of gearing

Cycloidal Profile



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Teeth Profiles satisfying the law of gearing

Cycloidal Profile

Properties of Cycloidal Profile

- In case of cycloidal teeth, the pressure angle varies from the maximum at the beginning of the engagement to zero when the point of contact coincides with pitch point and then again increased to a maximum in the reverse direction. It results in changing bearing reactions at the support.
- These gears must be operated at exactly the correct centre distance.
- Since cycloidal teeth are made up of the two curves, it is very difficult to produce accurate profiles. This has rendered this system obsolete.

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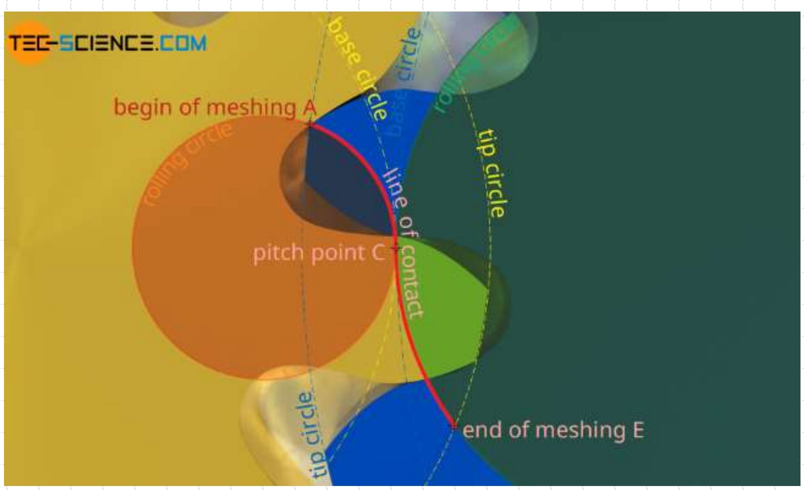
Meshing Cycloidal Gears



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