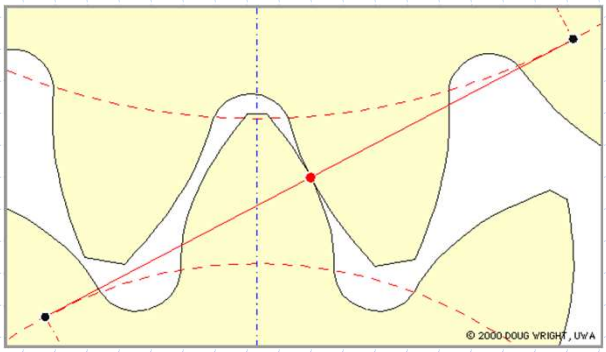
Path of Contact

Path of contact. It is the path traced by the point of contact of two teeth from the beginning to the end of engagement. Its length is called *length of path of contact*.



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Path of Contact

Length of Path of Approach

$$CP = CF - PF = \sqrt{R_a^2 - R^2 cos^2 \emptyset} - R \sin \emptyset$$

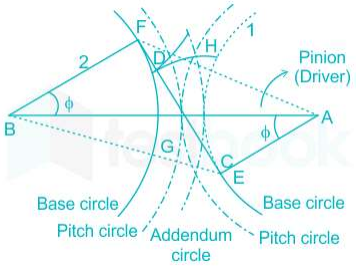
Length of Path of Recess

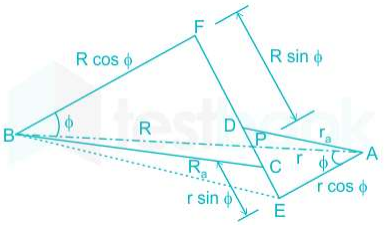
$$PD = DE - PE = \sqrt{r_a^2 - r^2 cos^2 \emptyset} - r \sin \emptyset$$

Length of Path of Contact

$$= CD = CP + PD$$

$$= \sqrt{R_a^2 - R^2 cos^2 \emptyset} + \sqrt{r_a^2 - r^2 cos^2 \emptyset} - (R + r) \sin \emptyset$$





- *Path of Approach depends on the dimensions of the driven wheel.*
- *Path of recess depends on the dimensions of the driving wheel (pinion)*

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Arc of Contact

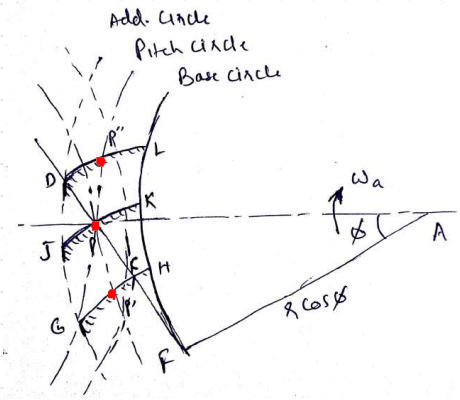
Arc of contact is the distance travelled by a point on either of the pitch circles of the two gears during the period of contact of a pair of teeth

GH: Driving involute at the beginning of contact

JK: Driving involute when the point of contact is at P.

DL: Driving involute at the end of engagement.

P′P″: Arc of contact = Arc of approach (*P′P*) + Arc of recess (*PP″*)



Arc of Contact

t_a = time of approach

Arc of approach = $P'P$ = Tangential velocity of P' × Time of approach

$$= \omega_a r \times t_a$$

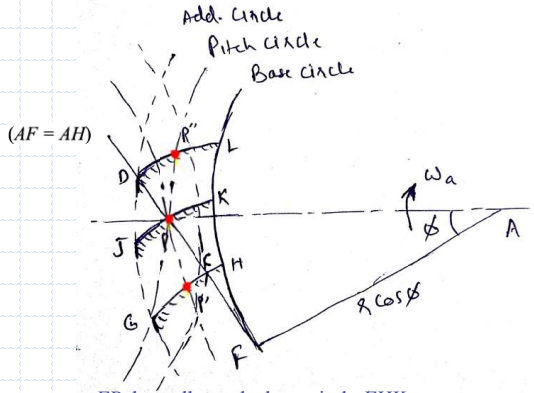
$$= \omega_a (r\cos\varphi)\frac{1}{\cos\varphi} t_a$$

$$= (\text{Tang. vel. of } H)\, t_a \frac{1}{\cos\varphi}$$

$$= \frac{\text{Arc } HK}{\cos\varphi}$$

$$= \frac{\text{Arc } FK - \text{Arc } FH}{\cos\varphi}$$

$$= \frac{FP - FC}{\cos\varphi} = \frac{CP}{\cos\varphi}$$



Arc *FK* = Path *FP* as *P* is a point on generator *FP* that rolls on the base circle *FHK* to generate the involute *PK*.

Similarly, Arc *FH* = Path *FC*

Arc *of approach* = $\dfrac{Path\ of\ approach}{\cos\phi}$

Arc of Contact

t_r = time of recess

Arc of recess = PP'' = Tang. vel. of $P \times$ Time of recess

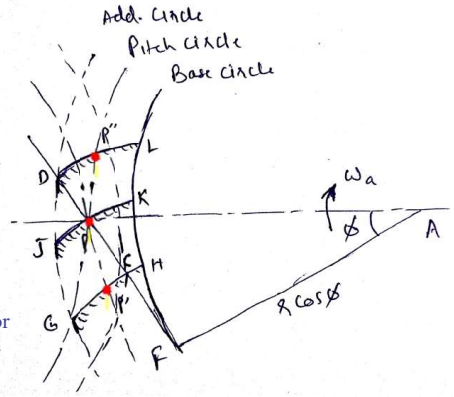
$$= \omega_a r \times t_r$$

$$= \omega_a (r\cos\varphi)\frac{1}{\cos\varphi} t_r$$

$$= (\text{Tang. vel. of } K)\, t_r \frac{1}{\cos\varphi}$$

$$= \frac{\text{Arc } KL}{\cos\varphi} = \frac{\text{Arc } FL - \text{Arc } FK}{\cos\varphi}$$

$$PP'' = \frac{FD - FP}{\cos\varphi} = \frac{PD}{\cos\varphi}$$



Arc FK = Path FP as P is a point on generator FP that rolls on the base circle FHK to generate the involute PK.

Similarly, Arc FL = Path FD

Arc *of recess* = $\dfrac{Path\ of\ recess}{\cos\phi}$

$$\text{Arc of contact} = \frac{CP}{\cos\varphi} + \frac{PD}{\cos\varphi} = \frac{CP + PD}{\cos\varphi} = \frac{CD}{\cos\varphi}$$

$$\text{Arc of contact} = \frac{\text{Path of contact}}{\cos\varphi}$$

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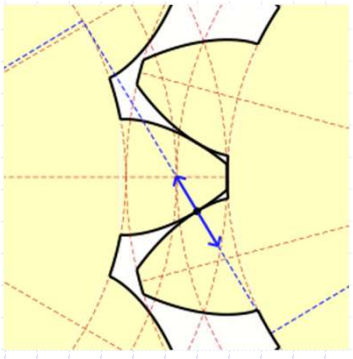
Contact Ratio

- The contact ratio of the gear set is defined as the ratio of length of arc of contact to circular pitch.
- This gives the average number of pairs of teeth in contact at a time.
- If contact ratio is more, the strength of gear set will be more.
- Contact ratio should always be greater than one.

Contact Ratio, $n = \dfrac{Length\ of\ arc\ of\ contact}{P_c}$



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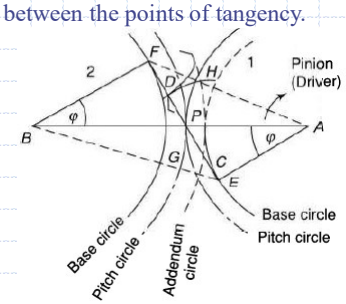
Interference

- The phenomenon when the tip of tooth undercuts the root of its mating gear is known as interference.
- The points E and F are called interference points.
- Interference may be avoided if the path of contact does not extend beyond interference points.
- The limiting value of the radius of the addendum circle of the pinion is *AF* and of the wheel is *BE*.
- Interference may only be avoided if the point of contact between the two teeth is always on the involute profile of the teeth. In other words, interference may only be prevented, if the addendum circle of the two mating gears cut the common tangent to the base circles between the points of tangency.



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Interference

- When interference is just avoided,

Maximum length of path of contact = EF

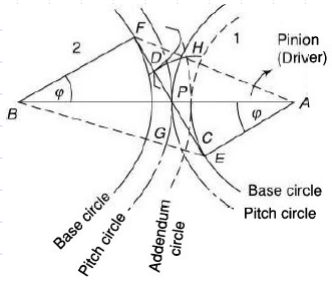
= EP + PF

Maximum length of path of approach, EP = $r \sin \emptyset$

Maximum length of path of recess, PF = $R \sin \emptyset$

$$\therefore EF = (r + R) \sin \emptyset$$

And Maximum length of arc of contact = $\frac{(r+R) \sin \emptyset}{\cos \emptyset} = (r + R)\tan \emptyset$



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Minimum no. of teeth to avoid Interference

Maximum value of addendum radius of the wheel to avoid interference = BE

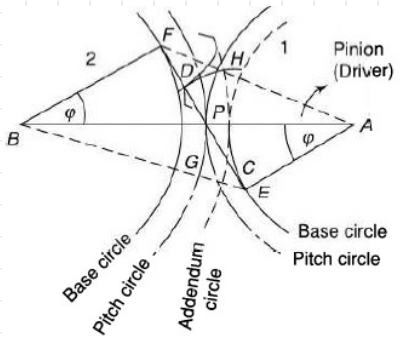
$$(BE)^2 = (BF)^2 + (FE)^2$$
$$= (BF)^2 + (FP + PE)^2$$
$$= (R\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2$$
$$= R^2\left[1 + \frac{1}{R^2}(r^2 + 2rR)\sin^2\varphi\right]$$
$$BE = R\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi}$$

Maximum value of addendum of the wheel
= (BE – Pitch circle radius)

$$a_{w\max} = R\left[\sqrt{1 + \frac{r}{R}\left(\frac{r}{R} + 2\right)\sin^2\varphi} - 1\right]$$

Now, $R = \frac{mT}{2}, r = \frac{mt}{2}$ and $G = \frac{T}{t}$ = Gear ratio

$$a_{w\max} = \frac{mT}{2}\left[\sqrt{1 + \frac{t}{T}\left(\frac{t}{T} + 2\right)\sin^2\varphi} - 1\right]$$
$$= \frac{mT}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1\right]$$



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Minimum no. of teeth to avoid Interference

Let Addendum = $a_w\, m$

This value of addendum must be less than the maximum value to avoid interference.

$$\frac{mT}{2}\left[\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1\right] \geq a_w m$$

$$T \geq \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1}$$

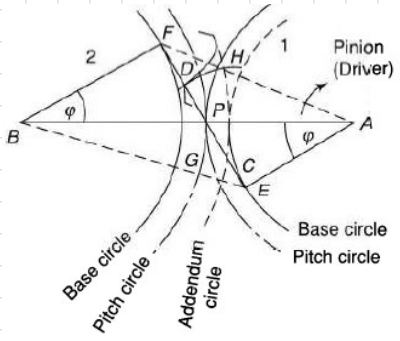
In the limit

$$T = \frac{2a_w}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1}$$

The minimum number of teeth on pinion is given by

$$t = \frac{T}{G}$$

For $a_w = 1$, $T \geq \frac{2}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\varphi} - 1}$



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Minimum no. of teeth to avoid Interference

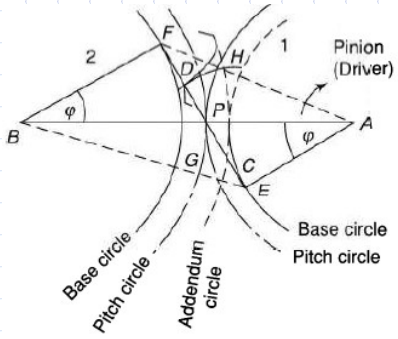
For $G = 1$,

$$T_{\min} = \frac{2}{\sqrt{1+3\sin^2\varphi} - 1}$$

For $\emptyset = 20^\circ$,

$$T_{\min} = \frac{2}{\sqrt{1+3\sin^2 20^\circ} - 1} = 12.31 \text{ or } 13$$

Thus for $G = 1$, $\emptyset = 20^\circ$, $a_w = 1$, the minimum number of teeth on each wheel must be 13 to avoid interference.



Maximum value of addendum radius of the wheel to avoid interference = AF

$$(AF)^2 = (r\cos\varphi)^2 + (R\sin\varphi + r\sin\varphi)^2$$

Maximum value of addendum of the pinion =

$$a_{p\max} = r\sqrt{1+\frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\varphi} - r = \frac{mt}{2}\left[\sqrt{1+G(G+2)\sin^2\varphi} - 1\right]$$

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Interference between rack and pinion

Maximum value of addendum of the rack to avoid interference = GE

Let Addendum of rack $= a_r\, m$

Where a_r is the addendum coefficient

This value of addendum must be less than the maximum value to avoid interference.

$$GE = PE\sin\phi = (r\sin\phi)\sin\phi = r\sin^2\phi$$

$$= \frac{mt}{2}\sin^2\varphi$$

To avoid interference,

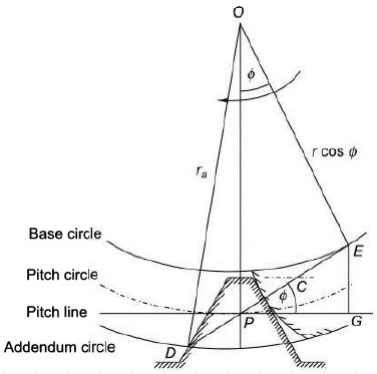
$$GE \geq a_r m \text{ or } \frac{mt}{2}\sin^2\varphi \geq a_r m \text{ or } t \geq \frac{2a_r}{\sin^2\varphi}$$

When a_r=1, i.e., for standard addendum,

$$t_{min} = \frac{2}{sin^2\emptyset}$$

For $\emptyset = 20^\circ$,

$t_{min} = 17.1$ or 18



$$\text{Path of contact} = CP + DP = \frac{\text{Add. of rack}}{\cos\varphi} + \sqrt{r_a^2 - (r\cos\varphi)^2} - r\sin\varphi$$

$$\text{Maximum path of contact to avoid interference} = DE = \sqrt{r_a^2 - (r\cos\varphi)^2}$$

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