MEC2120
Kinematics of Machines

Gear Trains

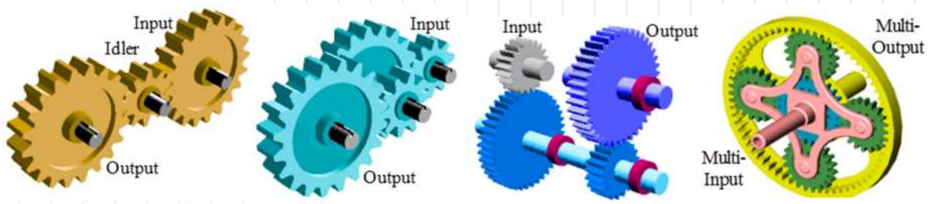
Gear Trains

When two or more gears are made to mesh with each other to transmit power from one shaft to another, the combination is called gear trains.

The nature of the train depends on velocity ratio and the relative position of the axes of shafts.

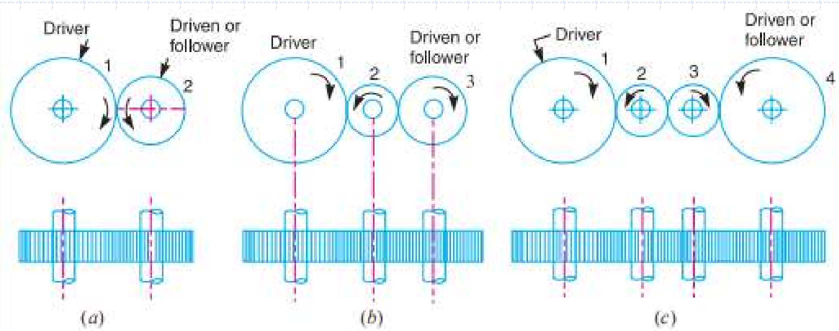
The gear train is classified into following types,

•Simple gear train.
•Compound gear train.
•Riveted gear train.
•Epicyclic gear train.

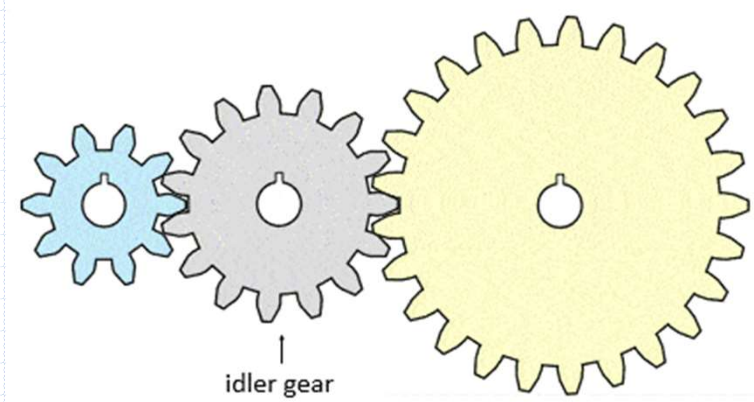


Simple Gear Train

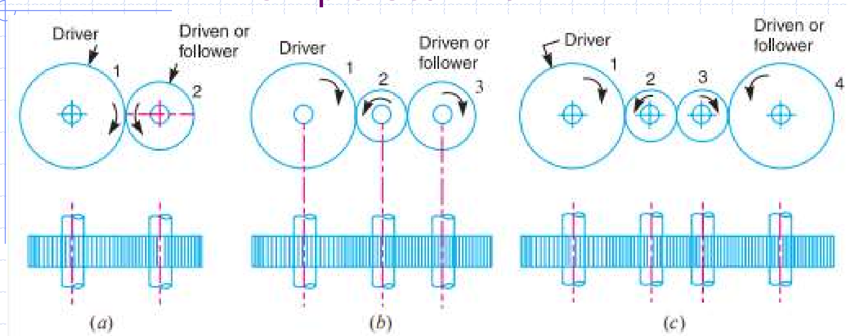
When there is only one gear on each shaft, it is known as simple gear train. The gears are represented by their pitch circles.



Simple Gear Train



Simple Gear Train



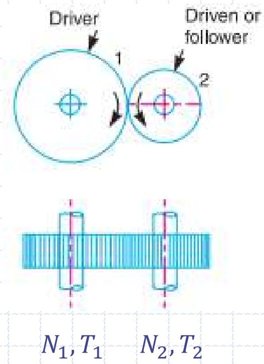
In a Simple Gear Train

- Two external gears of a pair always move in opposite direction.
- All odd numbered gears move in one direction and all even numbered gears move in the opposite direction.
- When the number of intermediate gears are odd, the motion of both the gears (i.e. driver and driven or follower) is like.
- If the number of intermediate gears are even, the motion of the driven or follower will be in the opposite direction of the driver.

Simple Gear Train

Speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth.

$$Speed\ Ratio = \frac{N_1}{N_2} = \frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$$



Train Value of a Gear Train is the ratio of the speed of the driven or follower to the speed of the driver.

$$Train\ Value = \frac{N_2}{N_1} = \frac{\omega_2}{\omega_1} = \frac{T_1}{T_2}$$

$$Train\ Value = \frac{1}{Speed\ Ratio}$$

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Simple Gear Train

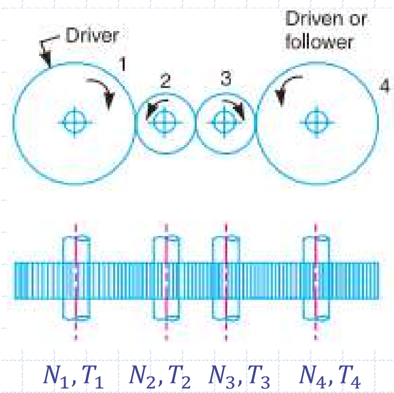
$$Speed\ Ratio = \frac{N_1}{N_4}$$

$$= \frac{N_1}{N_2} \times \frac{N_2}{N_3} \times \frac{N_3}{N_4} = \frac{N_1}{N_4}$$

$$= \frac{T_2}{T_1} \times \frac{T_3}{T_2} \times \frac{T_4}{T_3} = \frac{T_4}{T_1}$$

$$Speed\ Ratio = \frac{N_1}{N_4} = \frac{T_4}{T_1}$$

$$Train\ Value = \frac{N_4}{N_1} = \frac{\omega_4}{\omega_1} = \frac{T_1}{T_4}$$



- In a **Simple Gear Train**, the **Speed Ratio and Train value** are independent of the size and number of intermediate gears.
- These intermediate gears are called idle gears, as they do not effect the speed ratio or train value of the system.

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Compound Gear Train

When there are more than one gear on a shaft, it is called a Compound Gear train.

- For Gear Pair (Gear 1 and 2)

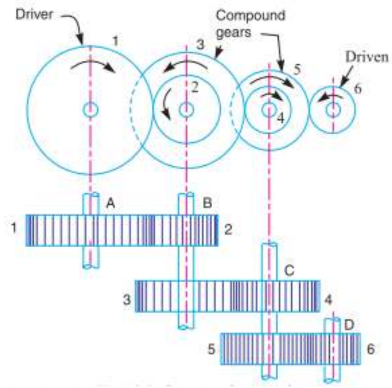
$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \quad \text{----(i)}$$

- For Gear Pair (Gear 3 and 4)

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \quad \text{----(ii)}$$

- For Gear Pair (Gear 5 and 6)

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \quad \text{----(iii)}$$



The speed ratio of compound gear train

$$\frac{N_1}{N_6} = \frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} \quad \left\{ \because N_2 = N_3, \; N_4 = N_5 \right\}$$

$$= \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$Gear\ 1\text{: } N_1, T_1$ $\quad$ $Gear\ 5\text{: } N_5, T_5$

$Gear\ 2\text{: } N_2, T_2$ $\quad$ $Gear\ 6\text{: } N_6, T_6$

$Gear\ 3\text{: } N_3, T_3$

$Gear\ 4\text{: } N_4, T_4$

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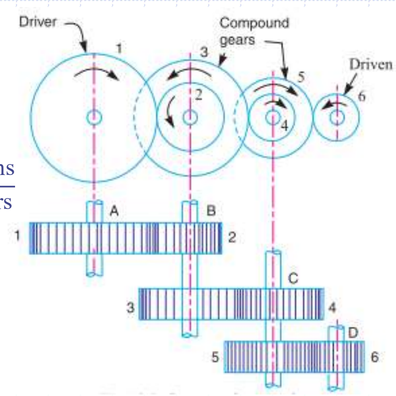
Compound Gear Train

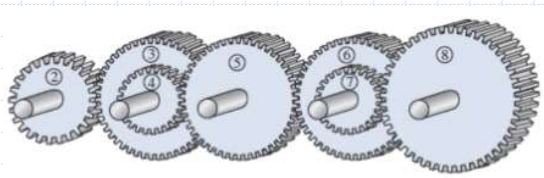
In a Compound Gear Train

$$Speed\ Ratio = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

$$= \frac{\text{Product of the number of teeth on the drivens}}{\text{Product of the number of teeth on the drivers}}$$

- The advantage of a compound train over a simple gear train is that a much larger speed reduction from the first shaft to the last shaft can be obtained with small gears.





$Gear\ 1\text{: } N_1, T_1$ $\quad$ $Gear\ 5\text{: } N_5, T_5$

$Gear\ 2\text{: } N_2, T_2$ $\quad$ $Gear\ 6\text{: } N_6, T_6$

$Gear\ 3\text{: } N_3, T_3$

$Gear\ 4\text{: } N_4, T_4$

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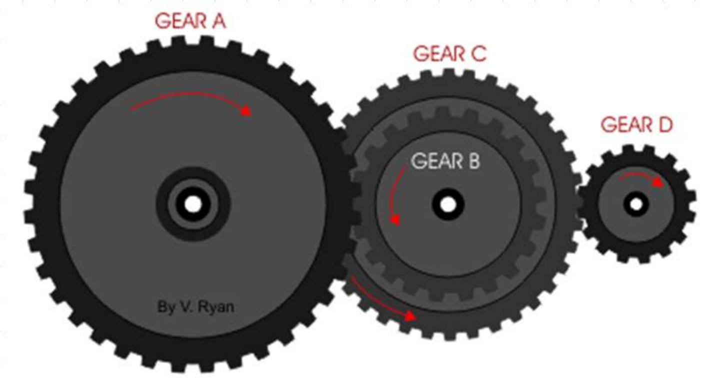

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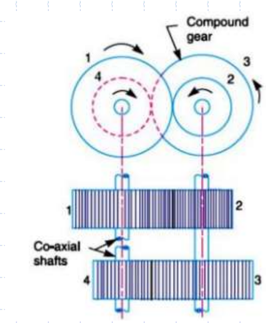
Reverted Gear Train

- When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as reverted gear train

$$C = r_1 + r_2 = r_3 + r_4$$

$$\frac{N_4}{N_1} = \frac{\text{Product of number of teeth on the drivers}}{\text{Product of number of teeth on the drivens}}$$

$$\frac{N_4}{N_1} = \frac{T_1}{T_2} \times \frac{T_3}{T_4}$$



- For the same circular pitch or module of all the gears

$$T_1 + T_2 = T_3 + T_4$$

- The reverted gear trains are used in automotive transmissions, lathe back gears, industrial speed reducers, and clocks.

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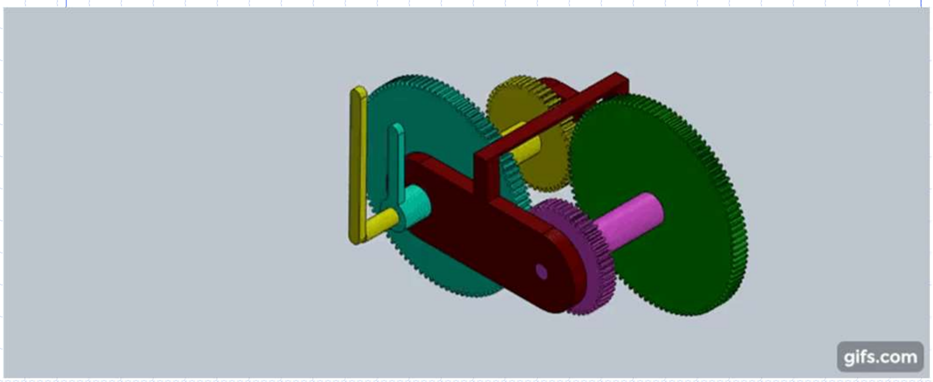

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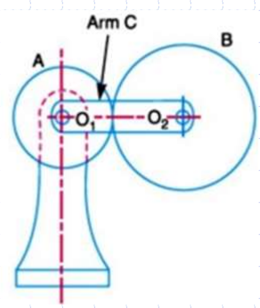
Epicyclic Gear Train

- In an epicyclic gear train, the axes of the shafts, over which the gears are mounted, may move relative to a fixed axis.
- The ***upon*** and ***around*** motion is called epicyclic motion (***epi*** **-** means upon and ***cyclic*** means around).
- The gear trains arranged in such a manner that one or more of their members move upon and around another member are known as ***epicyclic gear trains*** .
- The epicyclic gear trains may be ***simple or compound.***



- The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space.
- The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

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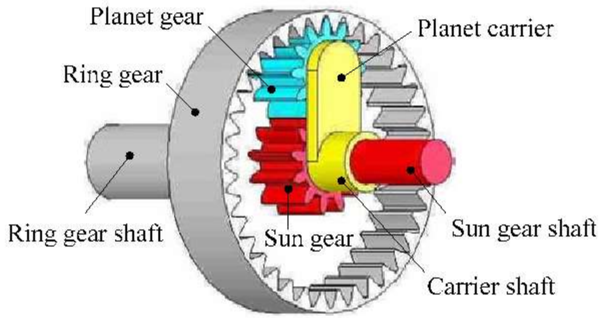
Epicyclic Gear Train



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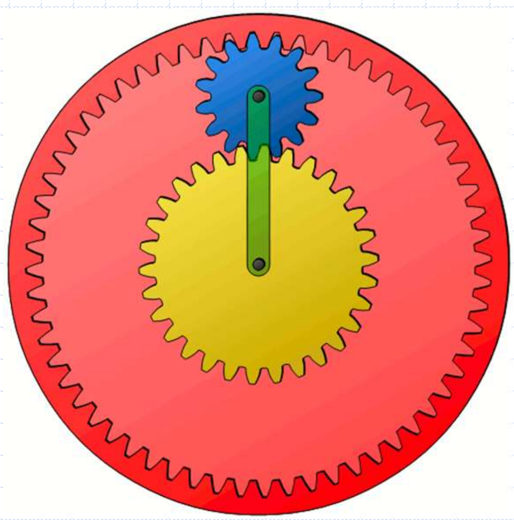
Epicyclic Gear Train



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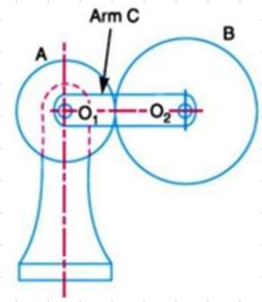
Analysis of Epicyclic Gear Train

Tabulation Method

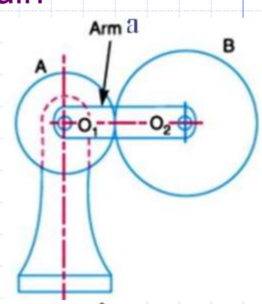
CONDITION OF MOTION	ARM C	GEAR A	GEAR B
The arm is fixed, Gear A rotates through +1 revolution	0	+1	-*Ta/Tb*
Arm fixed Gear A rotates through +*x* revolution	0	+*x*	-*x* × *Ta/Tb*
Add +*y* to all elements	+*y*	+*y*	+*y*
Total Motion	***y***	***x* + *y***	***y* - *x* × *Ta/Tb***



- Clockwise rotation is assumed as positive and Anticlockwise as negative.

Analysis of Epicyclic Gear Train

Problem: In an epicyclic gear train, an arm carries two gears A and B having 30 and 40 teeth respectively. If the arm rotates at 80 r.p.m. in the anticlockwise direction about the centre of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 240 r.p.m. in the clockwise direction, what will be the speed of gear B ?



Solution: Considering CCW rev. as + ve.

CONDITION OF MOTION	ARM a	GEAR A	GEAR B
The arm is fixed, Gear A rotates through +1 revolution	0	+1	-*Ta/Tb*
Arm fixed Gear A rotates through +*x* revolution	0	+*x*	-*x* × *Ta/Tb*
Add +*y* to all elements	+*y*	+*y*	+*y*
Total Motion	*y*	*x* + *y*	***y* - *x* × *Ta/Tb***

(i) Gear A is fixed, thus $y + x = 0$
Arm a rotates at 80 rpm, $y = 80$
$\therefore$ $x = -80$

Speed of the gear B, $y - \frac{3}{4}x$

$80 - \frac{3}{4} \times (-80) = 140 =$ rpm (counter-clockwise)

(i) Gear A revolves at 240 rpm clockwise,
$y + x = -240$
$\therefore \quad x = -80 - 240 = -320$

Speed of the gear B, $y - \frac{3}{4}x$

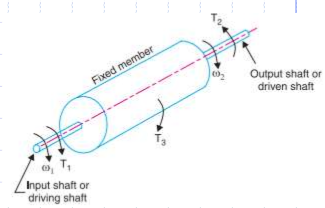
$= 80 - \frac{3}{4} \times (-320)$

$= 320$ rpm (counter-clockwise)

Torques in Epicyclic Gear Train

When the rotating parts of an epicyclic gear train have no angular acceleration, the gear train is kept in equilibrium by the three externally applied torques, viz.

1. Input torque on the driving member (T1),
2. 2. Output torque or resisting or load torque on the driven member (T2),
3. 3. Holding or braking or fixing torque on the fixed member (T3)



The net torque applied to the gear train must be zero.
In other words, T1 + T2 + T3 = 0

Let $\omega_1, \omega_2, \omega_3$ be the angular speeds of the driving, driven and fixed members respectively, and the friction be neglected, then the net kinetic energy dissipated by the gear train must be zero,
i.e.
$T_1\omega_1 + T_2\omega_2 + T_3\omega_3 = 0$...(iii)
But, for a fixed member, $\omega_3 = 0$
$\therefore T_1\omega_1 + T_2\omega_2 = 0$...(iv)
$\Rightarrow T_2 = -T_1 \times \frac{\omega_1}{\omega_2}$

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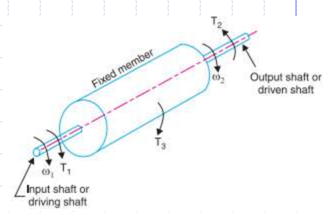
Torques in Epicyclic Gear Train

$$T_2 = -T_1 \times \frac{\omega_1}{\omega_2}$$

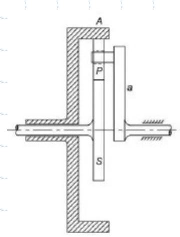
The above Equation shows that when input shaft (or driving shaft) and output shaft (or driven shaft) rotate in the same direction, then the input and output torques will be in opposite directions. Similarly, when the input and output shafts rotate in opposite directions, then the input and output torques will be in the same direction.



$$T_3 = -(T_1 + T_2)$$

$$T_3 = T_1\left(\frac{\omega_1}{\omega_2} - 1\right)$$

$$T_3 = T_1\left(\frac{N_1}{N_2} - 1\right)$$



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